The Construction of Mathematical Communication Schemes Based on Learning Styles

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Abstract

The purpose of this study was to describe the construction of students’ mathematical communication in solving problems based on students’ learning styles. This research method is qualitative-descriptive. The subjects in this research were three students, each of these students are theoretical, reflective, and pragmatic learning styles. The research instrument used was mathematics problems with a topic similarity. The data obtained were in the form of students’ work and short unstructured interviews. The data were analyzed based on indicators of problem-solving and mathematical communication. The results showed that students with theoretical learning styles could communicate the entire process of the problem-solving well. Students with a reflective learning style could show a good problem-solving process. Even so, mathematical communication is not very clear. Students with a pragmatic learning style had a poor performance in solving problems. The mathematical communication is also not very good.

Keywords
Communication, NCTM, mathematical ability, thinking style

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Introduction

Communication is an important part of daily life. Based on the National Council of Teachers of Mathematics, NCTM (2000), communication skill is one of the four basic skills that a person must possess to face a global society. Every individual is demanded to be able to communicate well and effectively so that the correct understanding and information are obtained among the individuals (Muqtada et al., 2018). Likewise in learning mathematics, each student has a way of communicating ideas in their own way. Mathematical communication is one of the five standard processes in learning mathematics (NCTM, 2000). Whereas the Program for International Student Assessment or PISA (OECD, 2013) makes mathematical communication one of the competencies of mathematical literacy, PISA states that the domain of mathematical literacy is the ability to analyze, reason, and communicate ideas effectively (OECD, 2013). This is also in line with the Regulation of the Minister of Education and Culture of the Republic of Indonesia No. 64 of 2013 which states that one of the competencies that students must have is to communicate mathematical ideas clearly and effectively.

According to Wilkinson, Bailey, and Maher (2018) College students’ communication skill is more complex and more abstract than the level below. College students have more tools and ways to communicate. Also, the explanation of college students’ thinking is more detailed and logical. College students are not only required to show results and explain the strategies they use but also to analyze, compare, and distinguish the meaningfulness, efficiency, and flexibility of the strategies used (Sür & Delice, 2016). Then secondary school students must be brave and not awkward in expressing their thoughts to others (Wilson, 2019). So, there are three important things in mathematics communication for secondary school students both written and oral namely complexity, logic, and courage to express opinions.

The quality of communication and students’ mathematical thinking are two things that are always related. This is supported by Triana and Zubainur (2019) which argue that the teacher is able to know, analyze, and evaluate mathematical thinking and strategies owned by students, when students are communicating with others. In addition, when students communicate both verbally and in writing, students make their thoughts and understandings clear to others (Kosko & Gao, 2017). A teacher must be able to know the thoughts of all students. This is because the teacher can use this information as a basis for determining further learning.

Students in a class have diverse characteristics. One way to see the characteristics of students is based on the tendency of learning styles they have. According to Danişman and Erginer (2017), learning style is defined as a tendency of cognitive characteristics, affective, and psychological behavior that are shown as indicators that appear relatively stable about the way students perceive, interact, and respond to their learning environment. Different learning styles are interesting to observe because each learning style has its way of constructing understanding. Also, students have their ways to communicate understanding. According to Lehman (2011) learning style is a method of someone’s best in terms of thinking, processing information and learning. According to Lehman (2011), there are at
least two important reasons that make communication in learning mathematics need to be improved among students. First, mathematics as language; mathematics is not just a tool to help think, for finding patterns, or solving problems, but mathematics is also an invaluable tool for communicating a variety of ideas, which means that it is a valuable tool to communicate ideas clearly, precisely and accurately. Second, mathematics learning as a social activity. In mathematics learning, the interaction between students, as well as teacher-student communication, is an important part of “nurturing children’s mathematical potential”. However, until now, the students’ mathematical communication skills have not received attention in learning enough.

**Literature Review**

**Students’ mathematical communication**

Mathematical communication is an event of oral or written dialogue that occurs in the classroom between teachers and students in conveying or explaining mathematical material, for example in the form of concepts, formulas, or mathematical problem-solving strategies. Pourdavood, Mccarthy, and Mccafferty (2015) explained that mathematical communication is very important. This is because with good mathematical communication students are be able to understand the purpose of existing problems; develop strategies in solving problems; use mathematical language such as mathematical symbols in solving problems; evaluate the concepts used; do calculations correctly; convey the results of the solution well, and convey mathematical ideas they have so that they can be understood by others. According to Disasmitowati and Utami (2017), mathematics communication skills include (1) the use of mathematical language which is presented in spoken, written, or visual form, (2) the use of mathematical representations presented in written or visual form, and (3) interpreting mathematical ideas, using mathematical terms or notations in representing mathematical ideas, and describing mathematical relationships or models. Additionally, Susanto (2013) explains that the indicators of students’ mathematical communication skills in mathematics learning are (1) the ability to express mathematical ideas through oral and written, also be able to demonstrate and describe them visually and (2) he ability to understand, interpret, and evaluate mathematical ideas both orally, in writing, and in other visual forms.

**Learning style**

Students in a class have various characteristics. One way to see the characteristics of students is based on their learning style tendencies. According to Danişman and Erginer (2017), learning styles are defined as a tendency for cognitive, affective, and psychological behavioral characteristics that are shown as indicators or markers that appear relatively stable about the way students perceive, interact, and respond to their learning environment. Different learning styles are interesting to observe because each learning style has its own way of constructing understanding. In addition, students also have their own way of communicating their understanding. According to Lehman (2011), learning style is a person’s best method in terms of thinking, processing information, and learning.
Based on the explanation above, the purpose of this research is to describe the mathematical communication process that occurs based on the thinking style of each student. Researchers hope that the results of this research can provide additional insight for academics and teachers in schools related to forms of mathematical communication carried out by students. In addition, the researcher also hopes that the results of the research can be used by the teacher to become a foundation in understanding the characteristics of mathematical communication given by students.

Methodology

The research method that has been used was a general qualitative because the purpose of this research was to describe the mathematical communication constructs carried out by students when solving problems. This type of qualitative research was chosen, considering that the data collected by researchers were from students' work (Creswell, 2014). The data analysis was carried out not based on the results of right and wrong, but the process of mathematical communication that occurred, so the approach used by researchers was a descriptive approach (Hake, 2007). Thus, researchers can construct a mathematical communication process that occurs in students when solving problems. The research subjects were 35 students taken randomly. The whole subjects were divided into 3 categories based on their learning styles, namely, (1) theoretical learning style subjects, (2) reflective learning style subjects, and (3) pragmatic learning style subjects. The research instrument used was mathematical problems about similarity as follows:

1. Work through the questions carefully and write down the problem-solving plan.
2. Write each step of your answer if necessary, prove the truth of each step
3. Mr. Tono is the manager at Rasa Restaurant. He advertises the dining area with isosceles trapezoid-shaped boards. The board is mounted on a three-legged wooden frame like the picture on the side. The length of wood used for one foot is 150 cm. The distance between the feet that stepped on the ground surface is 80 cm. Determine the length of the horizontal section of the board.

After the research subjects were given a test item, the results of the work in each category were analyzed. For each category, research subjects were selected for discussion and a mathematical communication scheme was made. To support the results of the study, researchers also conducted interviews with subjects. To simplify the process of analyzing research subjects, a ST code was given for theoretical learning style subjects, SR for reflective learning style subjects, and SP for pragmatic learning style subjects. Mathematical communication indicators used to analyze the results of the subject’s work were as follows (Hekimoglu & Sloan, 2015).
Table 1. Mathematical communication indicators

<table>
<thead>
<tr>
<th>Problem-Solving Stages</th>
<th>Indicators</th>
<th>Sub-Indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding the problem</td>
<td>Organize and consolidate mathematical thinking through communication</td>
<td>Write things that are known to be a problem using words</td>
</tr>
<tr>
<td></td>
<td>Use mathematical language to present mathematical ideas appropriately</td>
<td>Use mathematical symbols when writing information about problems.</td>
</tr>
<tr>
<td></td>
<td>Make a picture or illustration using facts about the problem.</td>
<td></td>
</tr>
<tr>
<td>Devising plan</td>
<td>Organize and consolidate the mathematical thinking through communication</td>
<td>Write the problem-solving plan</td>
</tr>
<tr>
<td></td>
<td>Use mathematical language to present mathematical ideas appropriately</td>
<td>Use mathematics terms while making problem-solving plan</td>
</tr>
<tr>
<td>Carrying out the plan</td>
<td>Communicate mathematical thinking coherently and clear to friends or teacher</td>
<td>Write the problem-solving and reason for each step</td>
</tr>
<tr>
<td></td>
<td>Analyze and evaluate others’ mathematical ideas and strategies</td>
<td>Write the mathematical ideas for commenting others’ answer</td>
</tr>
<tr>
<td></td>
<td>Use mathematical language to present mathematical ideas appropriately</td>
<td>Use mathematical terms while writing the problem-solving and reason for each step</td>
</tr>
<tr>
<td></td>
<td>Use mathematical symbols while writing the problem-solving and reason for each step</td>
<td></td>
</tr>
<tr>
<td>Looking back</td>
<td>Communicate mathematical thinking coherently and clear to friends or teacher</td>
<td>Change the mathematical symbol to the problem situation to write a conclusion</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Write conclusions and reasons for solving problems.</td>
</tr>
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</table>

This Mathematical Communication Indicator is the result of the indicator development described by Hekimoglu and Sloan (2015). Mathematical communication indicators in this research will be used as a basis to describe the mathematical communication process that occurs in students. The indicators in Table 1 are also the integration between problem management and mathematical communication. The data analyzed is the result of the work of students who have worked on the problem understanding given.

**Ethical consideration**

This study involved students in the primary school teacher education study program. The identity of the research subject is protected, including name, age, and the institution
where he belongs. Research subjects were also asked to perform the research procedure as previously described.

**Findings and Discussion**

Researchers carried out research in the elementary school teacher education study program. First, the researcher gave questions about the similarity to 35 students who were randomly drawn. Based on the results of the correction, only 17 students could answer the questions correctly, while the other 18 could not solve the questions. In addition, the results of student work also provided data related to student learning styles which are divided into 3 categories of thinking styles according to Honey-Mumford (Lehman, 2011). (1) Theoretical learning style, with the following considerations: (a) students like to adopt and integrate all of his observations into his frame of mind. This causes students to see how many observations are linked to each other; (b) students add a new learning into the existing framework by questioning and assessing possible ways so that the new information may be in accordance with the framework that students previously understood; (c) students have systematic thinking. The students often feel anxious in facing a problem until students get to the root of the problem; (d) students do not like anything that is subjective or ambiguous; (e) students usually use a problem-solving approach, a logical approach, or a step-by-step approach. (2) Reflective learning style with the following considerations: (a) students prefer to study behind and observe everything; (b) students like to gather as much information as possible before making a decision; (c) students always “see before he acts”; (d) students like to monitor the big picture, including previous experiences and the views of others. (3) Pragmatic learning styles with the following considerations: (1) students like to find out and make something using new ideas; (2) students look for practical implications of a new idea or theory before making an assessment; (3) students will take a look at something that has proven successful. However, if this does not work, students will spend less time analyzing the failure. After the results of the entire subject are checked, the results of the subject’s work can be seen in the following Figure 2.

**Figure 2. Subjects’ results**
Based on Figure 2, there were 15 subjects with theoretical learning styles, where 8 subjects answered correctly, and 7 subjects answered incorrectly. For the subjects of reflective learning style, as many as 4 subjects, where 2 subjects answered correctly and 2 subjects answered incorrectly. For pragmatic learning style subjects, there were 16, where 7 subjects answered correctly and 9 subjects answered incorrectly. The following discussion will choose 1 subject for each category randomly. So that there will be 3 research subjects that will be analyzed.

Subject ST (theoretical learning style)

Understanding the problem, the subject ST communicated the understanding related to the problem by making a picture that is equipped with information as shown in Figure 3. ST shaded the middle part of the triangle's building and then ST wrote that the picture “isosceles trapezoid board”. When the intent of the writing was confirmed, the picture made by ST is an illustration of the front of the real wooden frame.

P : What do you mean by the illustration you made?
ST : That's a wooden stand, sir, so the signboard is shaped like a trapezium, then placed on wood shaped like isosceles triangle. There are triangles in front and some behind.

Figure 3. ST's Problem-Solving Stages

Based on Figure 3, it also appears that ST wrote what is known by drawing three triangles of isosceles coinciding. ST gave the known measure of the sides of a triangle. It can be seen that ST mastered the concept of shape. This can be seen from the measure of the length of
the side written down accordingly. ST was also able to construct trapezoidal structures formed by cutting lines in the problem. According to Maulyda, Rahmatih, Gunawan, Hidayati, and Erfan (2020), making lines to form a new shape is one effective way to solve problems about shape. Cristobal and Lasaten (2018) also state that good visual representation ability is needed to understand pictorial problems, especially if the problem is accompanied by a contextual narrative. Besides, ST also gave a name (symbol) to facilitate the interpretation of the image made. From what ST wrote, it was also seen that ST understood what was asked. Based on this, ST was said to understand the problem given.

**Devising plan**, ST wrote a problem-solving plan by dividing the triangle shape into 3 parts, they are 1 triangle and 2 trapezoid. Then ST made line AJ to facilitate the preparation of plans. ST wrote the problem-solving plan well, where ST divided the horizontal lines DE and FG into 2. Conceptually, the problem-solving plan done by ST was to find the length of the sides "a" and "b". The mathematical model created by ST shows that he was able to connect the concepts of similarity and the ability of his number sense. Sukoriyanto, Toto, Subanji, and Tjang (2016) state that the number sense is a person’s ability to be sensitive to mathematical numbers. After that, each of the DH and FI lengths shown in Figure 3 that ST wrote its plan to determine the DH and FI Length using the congruence formula. According to Maulyda, Hidayanto, and Rahardjo (2019), making lines or new information in a solution shows that the subject has a good understanding of the concept.

**Figure 4. ST’s stage of devising plan**

Also based on Figure 4, ST did not write down the reasons for devising the plan. When the researchers confirmed this, ST mentioned that the triangle was congruent because all the corresponding angles were equal. When the researchers asked for a reason by emphasizing
the illustration of the problem, ST mentioned a reason that was not right, that is because in a triangle in which there are other triangles with their sides attached, then all measure of the angles are equal. The following quote from the interview transcript.

P : Well, based on the illustration about how you can be sure that the measure all angles are the same?
ST : Yes, because Sir, in the triangle there is a triangle like this (pointing to the triangle image T1 made), it must be the same angular size. The side is also stuck there.

Carrying out the plan, ST carried out a plan that had been prepared as shown in Figure 4. First, ST looked for DH values, by using the concept of similarity. ST compared the right triangle DHA and BJA. After that, ST calculated the known side length using the comparison rule. With the same steps, ST determined the value of FI so that ST had succeeded in determining the value of DH = 16 cm and FI = 24 cm.

Figure 5. ST’s carrying out the plan stage

Based on Figure 5, it also appears that the ST’s mathematical communication was good enough. ST’s writing structure was easy to understand and the calculation was also correct. Even so there were still shortcomings in ST’s writing, which was not given the length unit. ST only wrote “16” on the final result of the answer, even though the purpose of the number is 16 cm which represents the length of DH. According to Gorgorió and Planas (2015), this kind of mistake is often found in individuals who work on mathematics problems. This error is suspected to occur because the subject is in a hurry in solving the problem, or the lack of strength in the concept of the length shape. When researchers tried to dig deeper into ST’s understanding, ST stated that it should be given a unit for each step in writing the comparison. The following quotes from the interview.

P : What do you mean with 16?.
ST : 16 is the answer, Sir. The length of DH is 16.
P: There is no unit?
ST: Ah, yes. I'm in hurry solving the problem, Sir.
P: Is it okay or there is no unit in your final answer?
ST: It is a must, Sir.
P: Why?
ST: Yeah, we should be clear, Sir. 16 is the length of DH and we should write the unit too.

Figure 6. ST’s final work

In general, ST had succeeded in carrying out the previous plan. ST could find the length of DH and FI well. The calculations done by ST were also correct. Furthermore, ST determined the values of \( a \) and \( b \) through a previously made mathematical model, namely \( a = 2 \times DH \) and \( b = 2 \times FI \). The results of these calculations are the length of DE and FG as the length of the board asked in the problem.

**Looking back**, ST performed the stage of looking back because ST wrote the concluding sentence of the completion of the answer. ST subjects wrote their conclusions in sentence form. ST wrote that the length of **line a** is 38 while **line b** is 48. When the researchers confirmed the reason, ST wrote the following sentences,

ST’s answer: So, what is the length of horizontal section of the trapezoid board?
T1: This is, Sir. The top side is 32 cm and the lower part is 48 cm (pointing out to the conclusion part)
P: Why?
T1: Because \( a = 32 \) and \( b = 48 \)

This shows that ST changed the mathematical symbol into the problem situation when writing the conclusions and solved the given problem.

**Subject SR (reflective learning style)**

**Understanding the problem**, figure 7 shows that the SR communicated his understanding related to the problem given only by making a picture that is equipped with information. However, in the absence of writing any information that is known to cause the image created did not show in details what information is given to the problem. SR added symbol \( a \) as length \( DF \) and symbol \( b \) as length \( FG \). However, in the answers given, SR did not communicate what is represented by the symbols \( a \) and \( b \). Also, SR wrote additional \( H \) symbols that are not used in the problem-solving stage.
Based on Figure 7, it is known that SR understood the problem being asked about the problem, namely “length of the board of the restaurant” even though it is not the same as what was written in the problem, which is “determine the length of the horizontal part of the board”. SR did not give a specific mark (shading) on the part of the drawing that acts as a questionable board in the problem. Figure 7 also shows that SR adequately understood similar topic. SR could illustrate information in questions and added symbols and long information of each according to the location that is informed. In this case, communicating mathematical ideas can be done in various ways, namely through symbols, images, diagrams, and other media that can describe the problem situation (Annizar, Sisworo, & Sudirman, 2018). Also, good curiosity will make someone try to complete the information needed before solving the problem (Oonk, Verloop, & Gravemeije, 2019). This was confirmed by the results of the interviews conducted. The results of interviews with SR showed that the subject understood what was known and what was asked by the questions. Although SR did not write it clearly, but the picture made by SR is a media that SR used to write what is known and what is asked. In this case, SR tended to use visual communication (pictorial).

Devising plan, mathematical ideas can be realized by using pictures, graphs, tables, and diagrams (Yuniara, 2016). Based on this, SR made a problem-solving plan that begins by making 2 pairs of similar triangles. First is the triangles $DFC$ and the $ABC$. The second is the triangles $EGC$ and $ABC$. SR provided a description of the length of the sides of each triangle that matches the information that has been made at the stage of understanding the problem. This is done to simplify the planning stage. However, Figure 8 shows that the SR did not write down the problem-solving plan clearly, so that more detailed confirmation was needed through interviews. The SR’s plans only provided information “for 2 triangles”.
followed by making two pairs of triangles without giving further information regarding the planned problem-solving to be carried out. Conceptually, the problem-solving plan done by SR is to find the lengths of the sides “” and “” by comparing the sides of a similar triangle.

**Figure 8. SR's stage of devising plan**

![Diagram showing SR's stage of devising plan](image)

*SR has written a plan for completion, but did not provide a reason for using the plan.*

*The plan made by SR is to determine the length of the DF line (value of a) and the length of the line EG (value of b) by comparing the corresponding sides.*

**Carrying out the plan,** figure 8 shows the implementation of the plan prepared by SR. First, SR determined the value of a by using the concept of similarity. SR compared the isosceles triangles $DFC$ and $ABC$. After that, SR calculated the known side length using the comparison principle. With the same step, SR determined the value of b. So that SR had managed to determine the value of $a = 32$ and the value of $b = 48$.

P : Tell me how do you get this calculation?
SR : I compare the triangle 1 and 2, so that I do the cross-multiplication, Sir.
P : Why do you compare those triangles?
SR : I use similarity formula. If the shape is similar then we just have to compare the sides.
P : why don’t you write the reason?
SR : I don’t this it’s necessary, Sir. The most important this is the Calculation.

**Figure 9. SR's stage of carrying out the plan**

![Diagram showing SR's stage of carrying out the plan](image)

|E-ISSN: 2580-5711| https://online-journal.unja.ac.id/index.php/irje/index | 424
Based on Figure 9, SR is known to not write down the problem-solving in full and is not accompanied by a reason at each step of completion. However, SR could use mathematical language and the calculation process in each stage of completion is written correctly. Answers to mathematical problems that are communicated in writing must be able to explain the truth of the mathematical statement at each step done (Pantaleon et al., 2018). This is also supported by the results of interviews that stated that SR understood the concept of congruence that he uses to solve problems. SR also understood the reason for using the concept that the shapes of triangles 1 and 2 were made to have a similar shape, so the concept of similarity could be used to determine the unknown side. Therefore, it can be concluded that SR could carry out plans.

**Looking back**, stages of looking back can be done by SR subjects. Conclusions are written by the completion that has been done. The conclusion given by SR is in the form of a picture showing the location of the values \( a = 32 \) and \( b = 48 \) with the length unit, that is cm. SR obtained complete information about the shape of the board in the form of trapezoid EGFD with \( EG = 48 \text{ cm} \) and \( DF = 32 \text{ cm} \). SR changed the mathematical language into a complete sentence structure as a conclusion. The conclusion sentence has been able to answer the question on this problem correctly, that is the length of horizontal section of the board, they are \( 32 \text{ cm} \) and \( 48 \text{ cm} \). Ayuningtyas, Mardiyana, and Pramudya (2019) states that conclusions are formed from the integration between understanding of the question and solving the problem being carried out. If the two can be connected, then the conclusion will produce the correct answer. Figure 10 below is the answer to the SR at the stage of looking back.

**Figure 10. SR’s stage of looking back**

Subject SP (pragmatic learning style)

**Understanding the problem**, the SP communicated the understanding related to the problem given only by making a picture that is equipped with information as shown in Figure 11. SP did not write what is known by using words. SP could write down the information of the problem given in the picture and be accompanied by providing symbols.
SP also did not write in detail what the symbol represented. Let $x$ be the length of the $CF$ and $y$ be the length of the $GE$. Also, Figure 10 also shows that $x$ and $y$ are not placed in the middle as a sign of $GE$ and $CF$. This can happen if someone has difficulty in transforming a sentence in a problem into an appropriate representation or transforming a sentence in the problem into formal mathematical language (Wang & Wang, 2018).

Figure 11. SP’s stage of understanding the problem

Furthermore, Figure 11 also shows that the numbers written by SP are incomplete with their proper length unit. Based on these explanations and also the results of the interview it can be concluded that at this stage the SP was said to be unable to comprehend the problem given. Detailed information in a task or problem with a mathematical context becomes important that applies as a reference in solving problems (Kosko, 2016). Complete information can help avoid mistakes when solving mathematics problems. Almost in line with SR, SP did not write what is known and what is asked about the problem explicitly. However, SR made an image visualization to understand the given problem. Based on Figure 10 it also shows that the SP’s mathematical communication ability is still weak, because SP did not write the units at each known length. Also, writing the symbols $x$ and $y$ as the name of the line to be sought, is not in the middle so that it might cause misinterpretation.

**Devising plan**, the stage of devising a plan carried out by the SP is to communicate the two points of the plan in writing. First, SP looks for $x$ and $y$ values. Second, solve the problem by dividing the triangle $ABD$ in Figure 11 into two similar triangles. However, the SP does not provide additional information regarding the plans drawn up so interviews were needed to confirm the purpose of the two points of the plan. Problem-solving steps that are written systematically can help the subject in the process of solving mathematical problems (Pantaleon et al., 2018). Some people will find it difficult to communicate written mathematical ideas and it will be easier to express mathematical ideas verbally (Triana & Zubainur, 2019). Figure 12 shows the devising plan stage of SP.
The interview results showed that the SP made a good plan but the way to communicate the results of thinking is not right. This can be seen from the writing of the phrase “divide into 2 triangles” without writing up how many triangles were made. This shows that SP is not good at communicating mathematical ideas. After being interviewed, it turns out that the SP understands what he wrote so that it can be concluded that at this stage, SP’s communication skills are not good.

Carrying out the plan, at this stage, SP carried out what was planned in the previous stage. Figure 13 shows the SP looking for $x$ and $y$ values using a comparison between the sides of triangles. The triangle used in this step is derived from the half of triangle $ABD$ in Figure 11, which is right triangle. In the stage of finding the values of $x$ and $y$, SP drew two triangles $AHD$ as in Figure 12 and then writes information by adjusting the triangle modifications made. For the first triangle $AHD$ (i), SP is not careful to write down the value of each of the intended line lengths and also without giving the exact unit length (i.e. suppose the value 90 is located exactly between the lengths of $AD$). Based on Figure 11, the SP was incorrect in putting the point $E$ and also the point between the $AD$ which should be the point $G$ so that the value 90 is the length of the $AG$. Previous research also found that most students can count, but cannot explain precisely the meaning of the numbers written down (Maulyda, Hidayati, Erfan, & Umar, 2020).

Likewise in the second triangle $AHD$ (ii), SP was not careful in writing the unit of length of each number that is written as information in the picture. Besides, SP also did not precisely place point $F$ and the point that lies between $AD$ should be point $C$. SP did not pay close attention to the naming of points in Figure 13. In general, this can occur due to SP being inaccurate in drawing each triangle $AHD$ which is half of the triangle $ABD$. This finding is supported by previous research which found that only a few subjects could express
mathematical ideas through images correctly (Tiffany, Surya, Panjaitan, & Syahputra 2017; Yuniara, 2016).

**Figure 13. SP’s stage of carrying out the plan**

Furthermore, Figure 13 also shows that SP is not careful in carrying out the calculation process to determine the values of \(x\) and \(y\). To determine the value of \(x\), SP did not start by providing a row that contains the initial stage of completion, i.e., \(\frac{AD}{AH} = \frac{GD}{Gsesuatu}\) and the final stage of completion is completed by obtaining the values of \(x\) and \(y\) are 16 and 24 consecutively. SP did not start by writing down the appropriate initial stages, i.e., \(\frac{AD}{AH} = \frac{CD}{Csesuatu}\) and final completion stage just ended by obtaining the value of \(y = 24\). Both of these results are incorrect, because both values are only half of the length of the \(GE\) and \(CF\) based on Figure 11. This can occur because when drawing the triangle \(ABD\) as a whole (Figure 11), SP put \(x\) incorrectly in the middle of \(GE\) and put \(y\) not right in the middle of \(CF\). So there is a misunderstanding because the stage of “understanding the problem” is not fully correct, then the “carrying out the plan” stage will be disturbed. According to King (2016), conceptual failure in completion is caused by misconceptions experienced by students and is mistaken for errors in making modeling. In some other cases, it was found that one could easily illustrate the answers, but there were errors in making mathematical models (Deignan et al., 2019). In addition, the results of the study of Sumaji et al. (2019) also shows that problems that arise in communicating the solution of mathematical problems can occur if students do not understand the characteristics of the structure that must be described.

**Looking back**, SP did not do the “looking back” stage. This can be seen from the overall answers given by SP. The answer is that there is no process for changing the result of the calculation into the appropriate situation in the problem; not checking the solution again; and also not writing a conclusion at the end of the answer. The absence of a re-checking process can lead to an error in the final result of solving the problem (Reuter et al., 2015).
Based on the results of the interview, SP also stated that he did not re-check the results of his work. This is because the time is almost over and he is in a hurry to gather the answers. In the interview results, it was also revealed that SP was not aware of his mistake, so SP indeed did not understand the problem given. Thus it can be concluded that the results of problem-solving done by SP on this problem were not correct and have not answered the questions from the given questions.

Conclusion and Recommendations/Implications

In the theoretical learning style, students can understand the problem well. This is shown by its ability to write things that are known and write the name or symbol on the problem mathematically. Students are also able to plan but still lack understanding of the essence of planning. Students are also able to solve the problem given by the problem-solving plan. Students write the conclusions of the answers according to the language of the problem. Relative learning style students understand the information in the problem well. This is indicated by the use of visual communication that appears when identifying information on a problem. Students do not write their completion plans well. Problem-solving is done well by students’ reflective thinking. Even so, the form of written communication shown is not very clear. Students can write conclusions that have been adjusted to the context of the problem. In pragmatic learning style, students choose to use written communication in the form of images and symbols. The form of communication is poorly written but the plan for problem-solving is good. Plans that are well organized are not well executed by students. Pragmatic learning style students also do not do the stage of looking back. The written answers are no longer checked whether they are appropriate or not.

Disclosure statement

No potential conflict of interest was reported by the authors.

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