



Elementary Students' Mathematical Reasoning Abilities in Solving Non-Routine Fraction Story Problems

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Abstract

Fraction is an important part of elementary mathematics, yet they often cause difficulties because students may have trouble interpreting fraction concepts and applying them in unfamiliar situations. These difficulties can also affect students' reason when solving non-routine word problems. This study describes the mathematical reasoning demonstrated by elementary school students when solving non-routine problems involving fractions. A descriptive qualitative approach was used with six fifth-grade students from Public Elementary School 109 Palembang. The participants were purposively selected to represent high, medium, and low levels of mathematical ability. Data was collected through three non-routine essay problems designed to examine three aspects of mathematical reasoning: making conjectures, performing mathematical manipulation, and drawing logical conclusions. Students' writing was analyzed alongside interview data to examine how they interpreted information, conducted mathematical procedures, and justified their answers. The finding indicates that students showed various levels of reasoning across the three aspects. Making conjectures was the most challenging aspect, particularly when students had to identify relevant information and determine an appropriate initial strategy. In contrast, drawing logical conclusions was more frequently demonstrated, although some conclusions were not adequately supported by the preceding reasoning. The findings suggest that difficulties in solving fraction problems were not limited to procedural errors but were also related to students' conceptual understanding and interpretation of the problem context. These results highlight the need for fraction instruction that gives students greater opportunities to explain, justify, and evaluate their mathematical reasoning.

Keywords: descriptive qualitative; elementary school students; fractions; mathematical reasoning; non-routine word problems



INTRODUCTION

Mathematics education is expected to help students use what they learn to make sense of situations, make decisions, and deal with mathematical problems arising in different contexts (Shahbari & Peled, 2017). Such expectations cannot be met through conceptual knowledge and procedural fluency alone. Students need to examine mathematical relationships, explain the reasoning behind their strategies, and support the conclusions they reach with appropriate mathematical evidence (Brodie, 2009). In this sense, mathematical learning involves not only acquiring mathematical ideas but also learning how to use those ideas when encountering problems that vary in form and context (Rahmawati et al., 2023). This role of reasoning has consequently received increasing attention in mathematics education and curriculum development. Hjelte et al. (2020) described mathematical reasoning as important for making sense of concepts, establishing connections among mathematical ideas, and using those connections in problem solving. Classroom instruction therefore needs to provide students with opportunities to articulate their thinking, relate different mathematical ideas, and apply their knowledge when faced with situations that are less familiar to them (Elfariana et al., 2024). Despite this emphasis, however, students' reasoning does not always develop as expected, and difficulties in demonstrating mathematical reasoning continue to be reported.

The emphasis placed on mathematical reasoning in mathematics education is not always reflected in students' ability to use mathematics effectively when solving problems. The situation in Indonesia provides an important context for this concern. In the 2022 PISA assessment, Indonesia recorded a mathematics score of 366, compared with an OECD average of 472 (OECD, 2023; Pradypta et al., 2025). Although PISA does not directly assess mathematical reasoning according to the framework used in this study, the result provides a broader indication of the difficulties Indonesian students face when applying mathematical knowledge to problems that may be unfamiliar to them. More specific evidence can be found in studies conducted in Indonesia. Students may be able to conduct familiar procedures yet have difficulty explaining why those procedures are appropriate or how they relate to the information presented in a problem (Nusantara & Pradypta, 2026). Such difficulties are particularly evident when students are asked to identify relevant information, formulate an initial conjecture, establish connections among mathematical ideas, construct an argument, and support a conclusion with evidence (Anggraini et al., 2023; Hendriana et al., 2018; Nurazizah & Zulkardi, 2022). Classroom instruction that gives considerable attention to established solution procedures may contribute to this situation by providing limited space for students to build conceptual understanding and develop independent reasoning (Nabila et al., 2024; Saputri et al., 2017). By contrast, opportunities to discuss mathematical ideas, respond to formative feedback, and explain the reasoning underlying a strategy can create more supportive conditions for students' reasoning (Smit et al., 2023). Contextual problems, including tasks that share characteristics with PISA items, can likewise invite students to use mathematics in meaningful situations and engage both numeracy and mathematical reasoning (Nusantara et al., 2026; Nusantara & Pradypta, 2026). These considerations suggest that mathematics instruction needs to give greater attention to the reasoning students construct while solving problems, rather than judging their learning primarily from the answers they produce.

These concerns are particularly important in fraction learning because fractions require students to reason about quantities and relationships in ways that differ from their experience with whole numbers. At the same time, fraction knowledge provides a foundation for later topics such as decimals, percentages, and ratios and is strongly associated with students' ability to reason and solve mathematical problems (Copur-Gencturk, 2021; Rodriguez-Martinez et al., 2023). Understanding fractions therefore involves more than learning computational procedures. Students need to coordinate representations and address relationships among fractional quantities, with number lines and unit fractions offering important support for developing this understanding (Bruce et al., 2023). Despite this need, fractions remain difficult for many students. In fraction comparison, for example, students may judge the numerator and denominator separately, applying whole-number reasoning instead of considering the magnitude represented by the fraction. This tendency, known as the natural number bias, can produce systematic errors in comparing fractions (D'Erchie et al., 2025). Difficulties are also found in fraction operations and in interpreting fractional quantities across different situations,

especially when students reproduce procedures they have memorized without understanding the concepts underlying those procedures (Edo, 2016). Research has therefore explored ways of giving students richer opportunities to construct fraction meaning. A systematic review by Mohamed et al. (2021) found that visual, concrete, and symbolic representations can support students' conceptual understanding, while the review by Chn et al. (2024) pointed to the value of contextual and interactive activities that incorporate varied representations. Fraction models and manipulatives may further help students connect abstract ideas with more interpretable representations (Wilkie & Roche, 2023). Understanding equivalence is also important because it allows students to recognize that different fraction representations can refer to the same quantity and provides a basis for subsequent mathematical thinking (Pedersen & Bjerre, 2021). These supports, however, do not necessarily remove students' difficulties. Kim (2024) reported that students may still struggle to coordinate representations and understand fractions as numerical quantities when tasks require conceptual reasoning rather than the application of familiar procedures. Such demands become more pronounced in non-routine fraction word problems, where the solution path is not immediately apparent. Students must determine which information is useful, consider possible approaches, relate the mathematical ideas to the situation, and explain the basis of their solution. Non-routine fraction word problems therefore provide a meaningful context for examining mathematical reasoning because they require students to draw on fraction concepts, representations, and reasoning simultaneously when constructing a solution.

Research on fraction problem solving has revealed that students' difficulties extend beyond understanding fraction concepts and performing the required operations. Sari et al. (2024) and Hadiyanti and Manurung (2025), for example, reported difficulties in applying fraction concepts to word problems, conducting mathematical operations, and formulating appropriate conclusions. Similar concerns arise in adaptive reasoning, particularly when students need to apply mathematical concepts, choose a suitable strategy, and explain the basis of their answers (Mugianto et al., 2021). At the same time, contextual problem-solving activities, including PISA-like tasks, can provide situations in which students have opportunities to engage in mathematical reasoning through meaningful mathematical contexts (Nusantara & Pradypta, 2026; Putri & Zulkardi, 2019). These studies also point to a limitation in judging students' reasoning solely from their final responses. A correct or incorrect answer does not necessarily reveal how students interpreted the situation, selected and used relevant information, conducted mathematical procedures, or arrived at their conclusions. Looking more closely at these aspects can provide a fuller account of the reasoning that occurs during problem solving. This view corresponds with Nurazizah & Zulkardi (2022), who characterized mathematical reasoning in terms of how students understand, manipulate, and apply mathematical concepts while solving problems. Building on this perspective, the present study examines the mathematical reasoning demonstrated by elementary school students as they solve non-routine word problems involving fraction comparison.

Against this background, the present study examines how elementary school students demonstrate mathematical reasoning while solving non-routine word problems involving fractions. The emphasis is placed on the reasoning evident throughout the solution process rather than on students' achievement considered only as an instructional outcome or a score on predetermined indicators. Particular attention is given to how students interpret the information in a problem, formulate conjectures, perform mathematical manipulations, and arrive at logical conclusions. Examining these aspects together is intended to provide a more detailed picture of the characteristics of students' mathematical reasoning in non-routine fraction problem solving and to identify aspects that may need further attention in fraction instruction. The findings may also offer an empirical point of reference for future research on learning trajectories intended to support students' mathematical reasoning.

METHOD

Research Type

This study employed a qualitative descriptive approach to describe elementary school students' mathematical reasoning in solving non-routine fraction word problems. This approach was selected because it enables researchers to gain a comprehensive understanding of students' reasoning processes through the analysis of written responses and semi-structured interviews, allowing the characteristics of their mathematical reasoning during problem solving to be described in depth (Creswell & Poth, 2018). Qualitative descriptive design has likewise been used in mathematics education research to investigate students' reason and construct mathematical arguments when working on contextual tasks (Nusantara et al., 2025). Details concerning the participants and their selection are provided in the next subsection.

Research Subjects

Data collection was conducted in July 2025 in Class V.C of Public Elementary School 109 Palembang. The class comprised 24 fifth-grade students who took part in the study. From this group, six students were purposively selected for a more detailed examination of their mathematical reasoning, with the selection intended to represent various levels of mathematics ability (Creswell & Poth, 2018). The selected students were distributed across three ability levels: two high-ability, two medium-ability, and two low-ability students. This classification was determined through the classroom teacher's professional judgment, considering the students' mathematics achievement, classroom assessment results, and performance during mathematics lessons. Two of the six participants, identified by the codes NP and AZ, were subsequently involved in semi-structured interviews. They were selected because their written responses showed distinctive characteristics of mathematical reasoning and because both students were able to explain the reasoning underlying their solutions. The interviews were used to obtain further information about students' reasoning, particularly aspects of their thinking that could not be fully interpreted from the written responses alone.

Research Procedures

The research procedures were organized into five sequential stages to ensure the systematic implementation of the study. These stages comprised preparation, selection of research subjects, instrument validation, data collection, and data analysis, as illustrated in Figure 1.

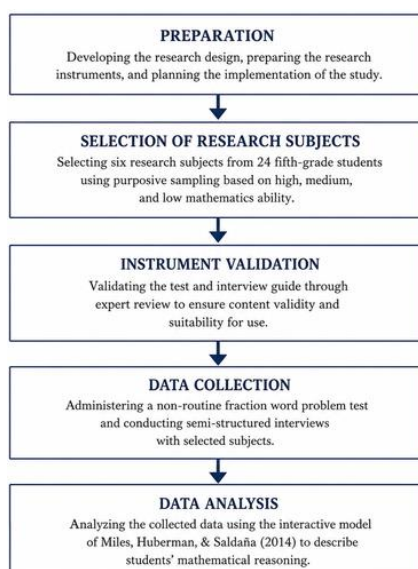


Figure 1. Research Procedures for Describing Students' Mathematical Reasoning in Solving Non-Routine Fraction Word Problems

As illustrated in Figure 1, each stage of the research was conducted sequentially to support the systematic collection of data required to address the research objectives. The data collection stage is described in greater detail in the following subsection.

Data Collection

Data was collected through a mathematical reasoning test and semi-structured interviews. The mathematical reasoning test consisted of three non-routine essay questions on comparing fractions designed to obtain data on students' mathematical reasoning in solving non-routine fraction word problems. Semi-structured interviews were conducted after the test to obtain a deeper understanding of students' thinking processes, solution strategies, and the reasons underlying each step of their problem-solving process.

Three dimensions of mathematical reasoning guided the construction of the test instrument: making conjectures, mathematical manipulation, and drawing logical conclusions, adapted from the framework proposed by Nurazizah & Zulkardi (2022). Together, these aspects were used to trace students' reasoning as they worked through the non-routine fraction word problems, beginning with their interpretation of the information and initial ideas for solving the problem, continuing with the mathematical procedures they employed, and ending with the conclusions drawn from their work. The indicators and their corresponding descriptions are provided in Table 1.

Table 1. Indicators of Mathematical Reasoning

Indicator	Descriptor
Making conjectures	Able to identify relevant information from the problem and formulate an initial mathematical conjecture or possible solution strategy based on the given information.
Mathematical Manipulation	Able to represent the problem mathematically and apply appropriate mathematical operations to obtain a solution.
Drawing Logical Conclusion	Able to draw logical conclusions that are consistent with the mathematical reasoning and solution obtained.

Prior to its implementation, the research instrument underwent content validation involving five fifth-grade teachers from Public Elementary School 109 Palembang. Two of the validators were specialists in mathematics education, whereas the other three were fifth-grade classroom teachers with relevant teaching experience. They reviewed the instrument in terms of the suitability of the mathematical reasoning indicators, the accuracy of the content, the formulation of the questions, the level of difficulty, and the clarity of the language. The comments and suggestions obtained during the validation were then used to revise the instrument before it was administered to the selected students.

Students' written responses were evaluated using a scoring rubric that reflected the extent to which each mathematical reasoning indicator was demonstrated. The scoring criteria used in the analysis are provided in Table 2.

Table 2. Scoring Rubric for Mathematical Reasoning Ability

Score	Reasoning Indicator
4	Perfect answer, the response is complete and correct.
3	The answer is correct, but the solution given contains one significant error
2	The answer is almost correct, but the solution contains more than one significant error
1	The answer is wrong; the solution is not completely resolved but contains at least one valid argument
0	The answer is wrong; the solution is based on process or argument that is wrong or contains no response at all

Data collection began with the administration of a mathematical reasoning test to the six research subjects. The test was completed individually using three non-routine essay questions on comparing fractions within 35 minutes. The use of non-routine essay questions was intended to provide students with opportunities to demonstrate their mathematical reasoning processes more

comprehensively while solving problems, as also adopted in previous studies on mathematical reasoning in fraction learning (Anggraini et al., 2023; Hadiyanti & Manurung, 2025). The three test items used in this study are presented in Table 3.

Table 3. Question Grids

No	Problem	Mathematical Reasoning Indicators Assessed	Characteristics as a Non-Routine Problem
1	Denni, Destri and Fika bought two ice lollies. Denni ate $\frac{1}{3}$ of the portion, Destri ate $\frac{1}{4}$ of the portion and Fika ate $\frac{3}{8}$ of the ice lolly. Sort the parts of the ice lolly they eat from smallest to largest.	Making conjectures, Mathematical manipulation, Drawing logical conclusions	Requires students to interpret the context, compare several fractions, determine an appropriate strategy, and justify the ordering rather than applying a memorized procedure.
2	Susanti received 30 tablets of medicine from the doctor, which she had to consume $1\frac{1}{2}$ tablets every day. In how many days will the medicine wear off?	Making conjectures, Mathematical manipulation, Drawing logical conclusions	Require students to interpret a mixed fraction in a real-life context, determine the appropriate operation, and justify the solution.
3	Pak Kardi has a plot of land that will be used as plantation land. He uses $\frac{1}{4}$ of the land to plant pineapples, $\frac{1}{3}$ to plant coffee and the rest will be planted with oranges. How much land will be planted with oranges?	Making conjectures, Mathematical manipulation, Drawing logical conclusions	Requires students to recognize the whole as one unit, determine the remaining fraction through reasoning, and justify the conclusion.

After all students' responses had been analyzed based on the mathematical reasoning indicators, two students coded NP and AZ were selected for semi-structured interviews. The interviews were conducted to verify the test results and to explore students' thinking processes, solution strategies, and the reasons underlying each step of their problem-solving process when solving non-routine fraction word problems.

Data Analysis

The data were analyzed qualitatively using the interactive analysis model proposed by Miles et al. (2014), which consists of data condensation, data display, and conclusion drawing and verification. Prior to the analysis, students' responses were assessed using a scoring rubric with a score range of 0–4 for each test item. Although the responses were converted into numerical scores, the analysis was primarily concerned with the evidence of mathematical reasoning shown in students' work rather than comparing their scores. After scoring, the responses were examined according to three indicators: making conjectures, mathematical manipulation, and drawing logical conclusions. For each indicator, a check mark (✓) was assigned when the student's response contained sufficient evidence to meet the corresponding descriptor, whereas a cross mark (X) was used when such evidence was absent. The resulting coding was used to describe the extent to which each student demonstrated the three reasoning indicators, with the overall results summarized in Table 3.

The written responses were then considered alongside the interview data to clarify the reasoning underlying students' solutions to the non-routine fraction word problems. The interviews provided additional information about the strategies students used, how they approached the problems, and why they produced particular responses in relation to each reasoning indicator. To strengthen the credibility of the interpretation, the two sources of data were compared through method triangulation. In this process, students' written work was examined together with their interview explanations so that interpretations of their mathematical reasoning were supported by complementary evidence from both sources.

RESULTS

The results draw on two sources of evidence: students' responses to the mathematical reasoning test and data from the semi-structured interviews. The analysis first examined the written work of the six participants to determine how the three mathematical reasoning indicators appeared across the three non-routine fraction word problems. The students' responses were then considered in relation to their mathematics ability levels high, medium, and low to identify similarities and differences in the characteristics of mathematical reasoning demonstrated by each group. Interview data from selected participants were then used to confirm and enrich the interpretation of the written responses by providing deeper insights into students' reasoning processes. A summary of the participants' performance across the three mathematical reasoning indicators and mathematics ability levels is presented in Table 4.

Table 4. Results of Analysis of Student Answer Sheets

Subject	Indicator	Question Number			Mathematics ability level
		1	2	3	
NP (S1)	1	✓	✗	✗	High
	2	✓	✓	✓	
	3	✓	✓	✓	
AZ (S2)	1	✗	✗	✗	High
	2	✓	✓	✗	
	3	✓	✗	✓	
ZA (S3)	1	✗	✗	✓	Medium
	2	✗	✓	✗	
	3	✗	✗	✗	
TA (S4)	1	✗	✗	✓	Medium
	2	✗	✓	✓	
	3	✗	✗	✓	
AI (S5)	1	✗	✗	✓	Low
	2	✗	✓	✓	
	3	✗	✓	✓	
MF (S6)	1	✗	✗	✓	Low
	2	✗	✓	✓	
	3	✗	✗	✗	

Note:

✓: Appear

✗: Not Appear

As shown in Table 4, the six participants demonstrated various levels of mathematical reasoning across the three indicators. The making conjectures indicator was demonstrated only three times across the six participants and three test items, making it the least frequently demonstrated indicator. In contrast, drawing logical conclusions were demonstrated thirteen times, whereas mathematical manipulation appeared ten times. Overall, high-ability students demonstrated more mathematical reasoning indicators than medium- and low-ability students. Nevertheless, variations in reasoning characteristics were observed across all ability groups, particularly in the making conjectures indicator. These characteristics are discussed in detail in the following sections.

Reasoning Ability in the First Indicator (Making Conjectures)

Table 4 shows that making conjectures was the least frequently demonstrated mathematical reasoning indicator among the participants. Of the six participants, this indicator appeared in only four responses across the three questions analyzed, consisting of one response in Question 1 and three responses in Question 3. Most participants experienced difficulties identifying relevant information

and formulating an appropriate initial conjecture before determining a solution strategy. These characteristics are further explained through the analysis of representative students' written responses.

High-Ability Students

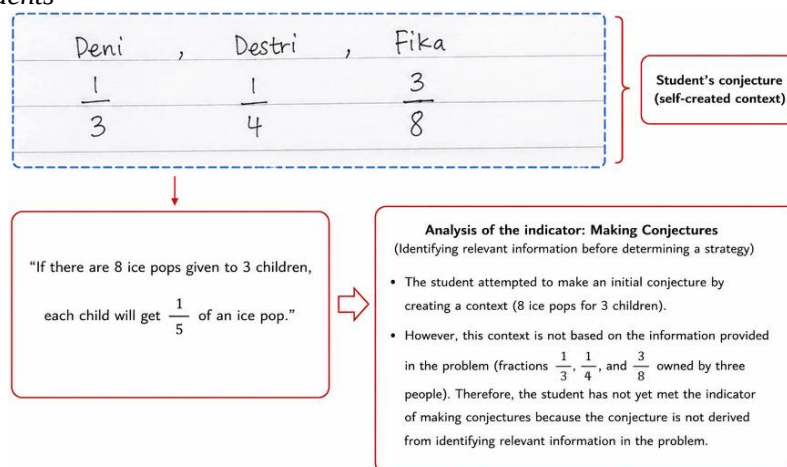


Figure 2. Written Response of a Student with High Mathematical Ability (NP) Demonstrating the Making Conjectures Indicator

Figure 2 shows NP's written response from the high-ability group to Question 1. Before determining a solution strategy, NP first identified the relevant information by recording the fractions associated with each person, namely $\frac{1}{3}$, $\frac{1}{4}$, dan $\frac{3}{8}$. This step indicates that NP recognized the essential information required to begin solving the problem. Therefore, NP fulfilled the making conjectures indicator by identifying relevant information and formulating an appropriate initial mathematical approach before proceeding to mathematical manipulation.

Medium-Ability Students

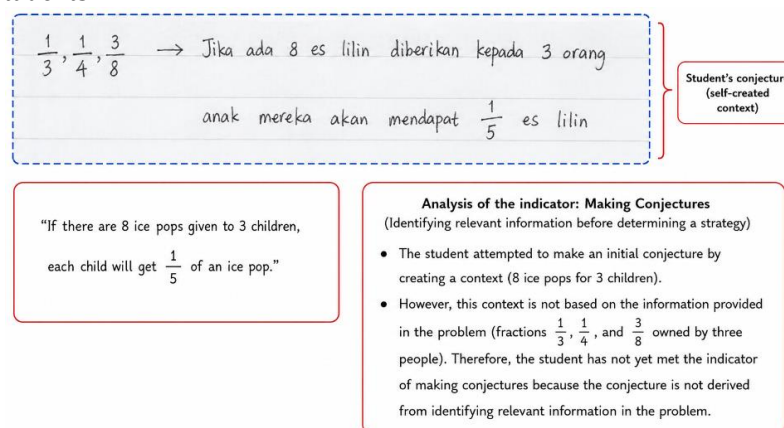


Figure 3. Written Response of a Student with Medium Mathematical Ability Demonstrating the Conjectures Indicator

As shown in Figure 3, the student with moderate mathematical ability attempted to formulate an initial conjecture by transforming the given information into a new context involving the distribution of eight ice pops among three children. Although the student demonstrated an effort to construct an initial conjecture, it was not based on the information provided in the problem but rather on a self-created context. Consequently, the student was unable to identify the relevant information needed to determine an appropriate solution strategy. Therefore, the making conjectures indicator was not

achieved because the initial conjecture was not derived from the mathematical information presented in the problem.

Low-Ability Students

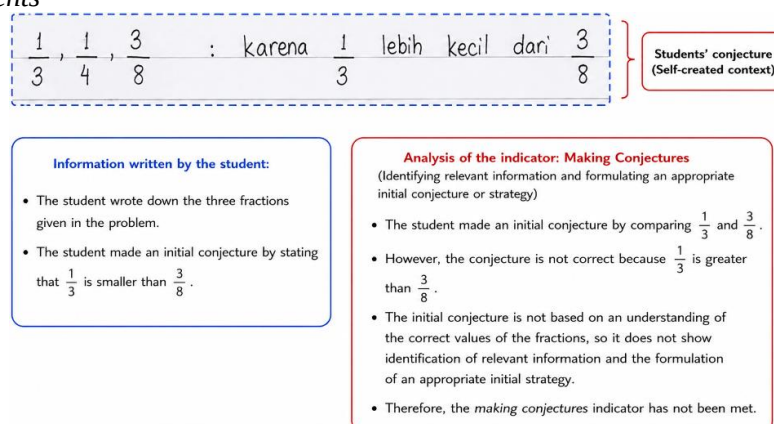


Figure 4. Written Response of a Student with Low Mathematical Ability Demonstrating the Conjectures Indicator

As shown in Figure 4, the student's response indicates an attempt to formulate an initial conjecture before determining a solution strategy. However, the conjecture was not based on an appropriate interpretation of the mathematical information provided in the problem, resulting in an incorrect comparison of fractions. This finding suggests that the student was unable to use the relevant information as the basis for formulating an appropriate initial conjecture. Therefore, the student did not achieve the making conjectures indicator.

Reasoning Ability in the Second Indicator (Mathematical Manipulation)

Table 4 shows that mathematical manipulation was the most frequently demonstrated mathematical reasoning indicator among the participants. Most participants were able to represent the problems mathematically and apply fraction operations across several questions. Nevertheless, variations were observed in mathematical representations and the appropriateness of the procedures used to obtain solutions. These characteristics are further illustrated through the analysis of representative students' written responses from different mathematical ability levels.

High-Ability Students

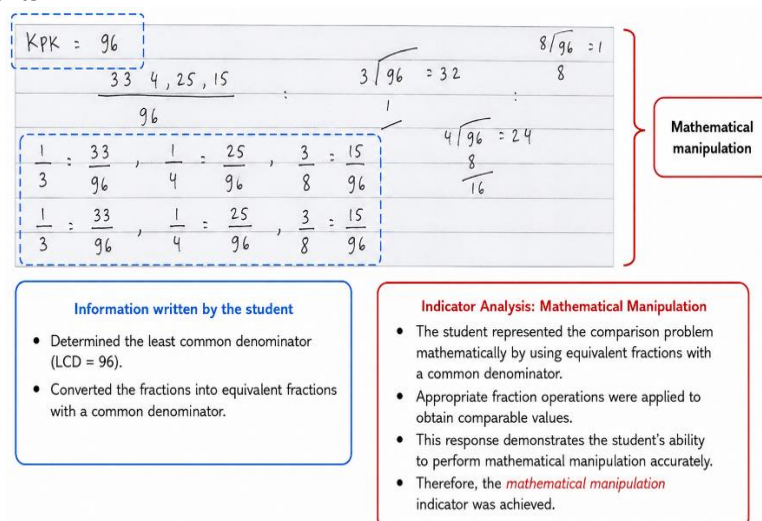


Figure 5. Written Response of a Student with High Mathematical Ability (NP) Demonstrating the Mathematical Manipulation Indicator

As shown in Figure 4, the student's response demonstrates the ability to represent the problem mathematically through an appropriate solution strategy. The mathematical manipulation performed reflects the student's ability to apply appropriate fraction operations systematically to obtain the correct solution. The student's response provides sufficient evidence of mathematical manipulation, as the problem was represented mathematically and the appropriate operations were then applied to obtain the solution.

In Question 2, the student with high mathematical ability also demonstrated the mathematical manipulation indicator through a different mathematical representation. The student's response is presented in Figure 6.

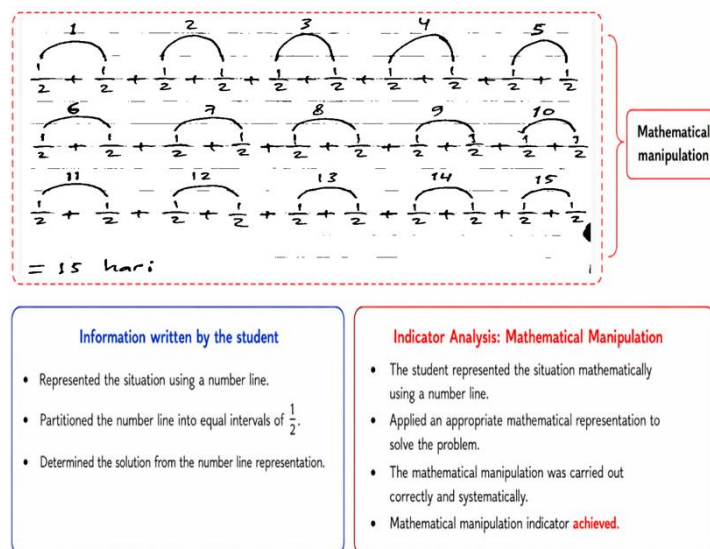


Figure 6. AZ Student's Answer to the Second Indicator from Question Number 2

AZ's response to Question 2 is shown in Figure 6. The written work contains a sequence of mathematical steps that reflects the use of mathematical manipulation, although the solution ultimately produced an incorrect answer. A semi-structured interview was subsequently conducted with AZ to clarify how the student understood and approached the problem. An excerpt from the interview is provided below.

- R : "Answer number 2, why can you get the results in 15 days?"
- AZ : "I got the answer from here miss (pointing to question number 2).
So, $1\frac{1}{2}$ of the medicine for a day means taking $\frac{1}{2}$ of the medicine once, right miss?"
- R : "Okay. Keep going?"
- AZ : "The number of medicines given was 30, then divided into 3 miss, so $\frac{1}{2}$ each was made into 30 according to the number of medicines. That is 15 days miss.
- R : "what about this?"
- R : "What about this?" Isn't there $1\frac{1}{2}$ medicines that Susanti takes a day?
- AZ : "I see. Oh, well, I am confused, Miss."
- R : " $1\frac{1}{2}$ equals how much?"
- AZ : "I don't know miss. We haven't studied at school, so we don't understand miss."

The interview revealed that AZ did not understand the meaning of the mixed fraction $1\frac{1}{2}$ and interpreted it as $\frac{1}{2}$. AZ also explained that the concept of mixed fractions was still unfamiliar because it had not yet been studied during classroom instruction. This finding suggests that AZ's difficulty was not primarily related to performing mathematical operations but rather to an incomplete conceptual

understanding of mixed fractions. Consequently, although AZ attempted to apply a systematic mathematical procedure, the reasoning process was based on an incorrect interpretation of the given information, leading to an incorrect solution. Nevertheless, Figure 3 shows that AZ demonstrated the mathematical manipulation indicator by attempting to represent the problem mathematically and follow a coherent solution procedure, even though the procedure did not produce the correct answer.

Medium-Ability Students

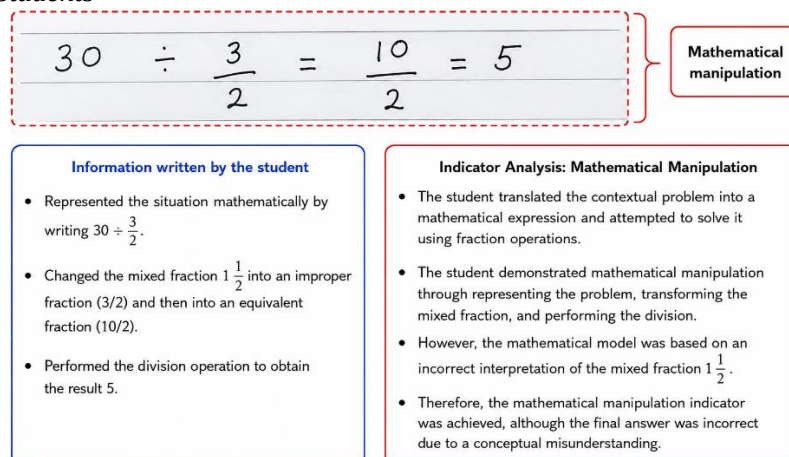


Figure 7. ZA Student's Answer to the Second Indicator from Question Number 2

Figure 7 illustrates how the student with medium mathematical ability approached the problem. The student first translated the information in the problem into a mathematical representation and then proceeded with fraction division. The written work indicates that the student was able to conduct the required mathematical manipulation, but the answer obtained was incorrect. The error can be traced to the initial mathematical representation, which was based on a misunderstanding of the mixed fraction. This interpretation was confirmed during the interview, when the student explained that the mixed fraction had been treated as an ordinary fraction because the concept had not yet been introduced in classroom instruction. Thus, the difficulty observed in this response was not mainly related to conducting the calculation, but to understanding and representing the fractional quantity described in the problem.

Low-Ability Students

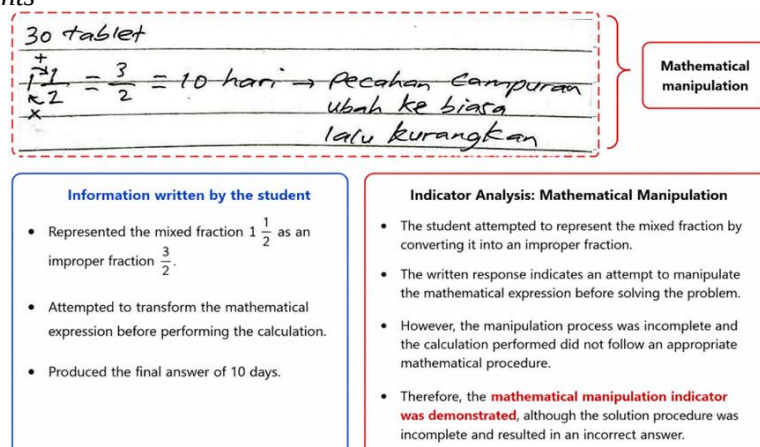


Figure 8. MF Student's Answer to the Second Indicator from Question Number 2

Figure 8 presents the response of the student with low mathematical ability. The student began by attempting to rewrite the mixed fraction as an improper fraction before proceeding with the

calculation. However, the transformation was not carried through correctly, and the subsequent calculation did not lead to an appropriate solution. The written response suggests that the student understood that the mixed fraction needed to be represented differently before the operation could be performed but had difficulty conducting the next steps in a consistent manner. The error therefore appears to involve both an incomplete understanding of the fraction representation and difficulties in executing the mathematical manipulation required to solve the problem.

Reasoning Ability in The Third Indicator (Drawing Logical Conclusion)

As presented in Table 4, evidence of the drawing logical conclusions indicator was found in responses from all three mathematical ability groups, although it was not demonstrated consistently by every student or across all questions. In general, students who showed this indicator were able to state a conclusion after working through the problem. The strength of their conclusions, however, depended on the quality of the reasoning and the correctness of the mathematical procedures that preceded them. The nature of these differences is illustrated further through representative written responses from students in the high-, medium-, and low-ability groups.

High-Ability Students

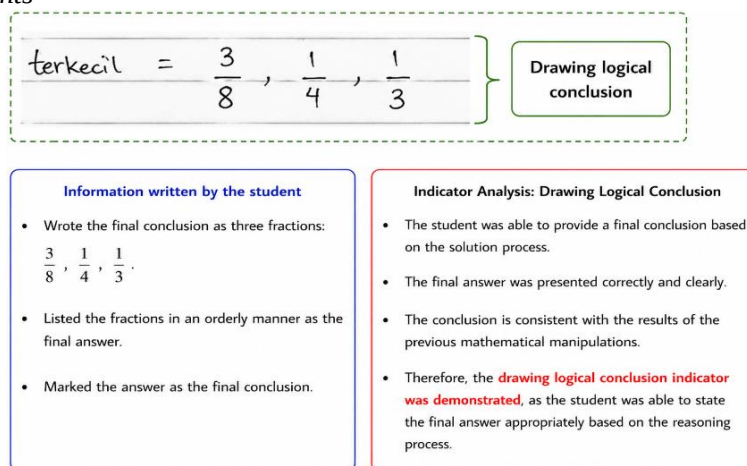


Figure 9. NP Student's Answer to the Third Indicator from Question Number 1

Figure 9 presents the response of a student with high mathematical ability who arrived at a conclusion that was consistent with the preceding solution. The final statement followed logically from the mathematical work shown in the response, indicating that the student was able to bring the results of the manipulation together into an appropriate answer. The response also suggests that the student understood the connection between the procedures used and the conclusion reached, rather than simply stating an answer without reference to the preceding mathematical reasoning.

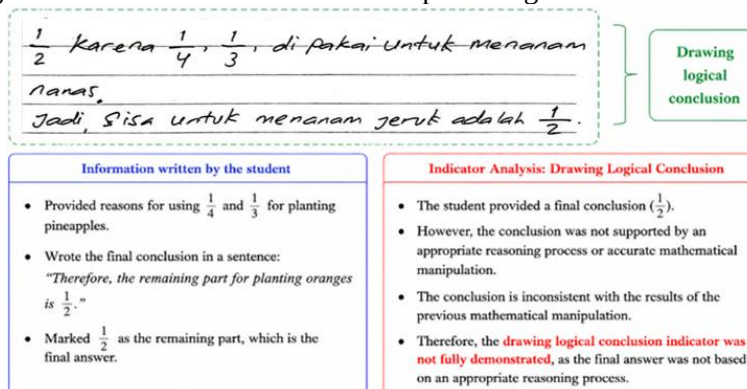


Figure 10. AZ Student's Answer to the Third Indicator from Question Number 3

Figure 10 illustrates that the student's written conclusion was influenced by the student's interpretation of the problem context. To gain a deeper understanding of the reasoning underlying this response, a semi-structured interview was conducted with AZ, as presented in the following excerpt.

- R : "How can you get answer $\frac{1}{2}$ for question number 3?"
 AZ : "From the miss question, there were already $\frac{1}{4}$ and $\frac{1}{3}$ in order so I answered $\frac{1}{2}$."
 R : "Why is that?"
 AZ : "Actually, I do not understand this problem. Why isn't information about the size of the land given? So, it is impossible to calculate it."

The interview indicated that AZ viewed the problem as incomplete because the total area of the land was not explicitly provided. As a result, the student did not consider the given land as a whole unit that could be represented mathematically and used in fraction operations. The difficulty, therefore, appears to have arisen primarily from how the student interpreted the information in the problem rather than from an inability to conduct the required calculations. Similar responses were identified in several other participants, indicating that making sense of contextual information is an important part of forming conclusions that are appropriately supported by mathematical reasoning.

Medium-Ability Students

Jadi, tablet obat itu habis dalam 5 hari.

Drawing
logical
conclusion

Information written by the student	Indicator Analysis: Drawing Logical Conclusion
- Wrote the final conclusion in a sentence: "Jadi, tablet obat itu habis dalam 5 hari." - Provided a final answer in the form of a complete sentence. - Marked the sentence as the final conclusion.	- The student was able to provide a final conclusion based on the solution process. - The final answer was presented correctly and clearly using a complete sentence. - The conclusion is consistent with the results of the previous mathematical manipulation. - Therefore, the drawing logical conclusion indicator was demonstrated, as the student was able to state the final answer appropriately based on the reasoning process.

Figure 11. ZA Student's Answer to the Third Indicator from Question Number 1

Figure 11 shows that the quality of the students' conclusion was closely related to the reasoning that preceded it. The student presented the final answer as a complete statement; however, the interview showed that the mixed fraction had been interpreted incorrectly, which led to an inappropriate mathematical representation of the problem. Thus, although the student was able to formulate a final conclusion, its validity was affected by the misunderstanding underlying the earlier stages of the solution.

Low-Ability Students

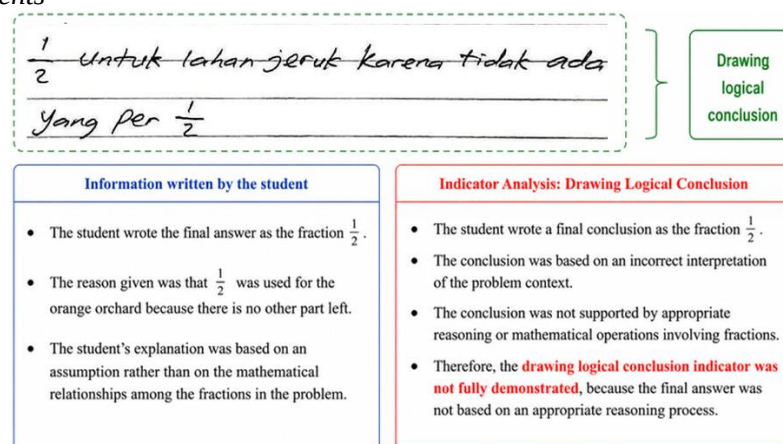


Figure 12. TA Student's Answer to the Third Indicator from Question Number 3.

Figure 12 shows that the students' explanation for the final answer was not based on the mathematical relationships between the fractions given in the problem. Instead, the student used assumptions about information that was not directly provided in the problem when arriving at the answer. This response indicates that a logically supported conclusion requires more than stating a final result; students also need to relate the information available in the problem through appropriate mathematical reasoning.

DISCUSSION

The results offer a closer view of elementary school students' reason when working on non-routine problems involving fractions. The discussion considers these findings through the three indicators examined in the study: making conjectures, mathematical manipulation, and drawing logical conclusions. The emphasis is not only on what students were able to do, but also on the factors that may explain the reasoning evident in their responses. To interpret these patterns, the findings are considered in relation to relevant previous research. These findings also offer implications for mathematics education, particularly in relation to theoretical perspectives and classroom practice.

Making Conjectures in Solving Non-Routine Fraction Word Problems

Among the three indicators examined, making conjectures was the aspect that students found most difficult. The written responses indicate that the difficulty arose mainly at the beginning of the solution process, when students needed to determine which information was relevant and make sense of the situation described in the problem. In several responses, students either made assumptions that were not supported by the given information or interpreted part of the context inaccurately. As a result, the reasoning that followed was built on an inappropriate starting point. The difficulties observed in this study therefore appear to involve both students' understanding of fraction concepts and their ability to interpret the problem situation before deciding how to proceed.

This result can be understood in relation to the way mathematical reasoning has been conceptualized in previous research. Brodie (2009) described reasoning as beginning with students' efforts to interpret and make sense of a mathematical situation before they engage in manipulation and justification. Shahbari & Peled (2017) likewise associated conjecturing with the process of relating information in a problem to mathematical ideas that can guide subsequent solution attempts. When this initial connection is not established appropriately, later reasoning may also be affected, as suggested by Nurazizah & Zulkardi (2022) in their study of contextual mathematical problems. Evidence from fraction word problems provides a similar indication. Hadiyanti & Manurung (2025) found that students' errors were often related to how they interpreted information contained in the problem situation, while Anggraini et al. (2023) emphasized the importance of identifying relevant information and forming an initial mathematical idea before proceeding to formal solution procedures.

Viewed together, these studies provide a possible explanation for why making conjectures emerged as the most demanding indicator in the present study: an inaccurate interpretation at the outset can shape the direction of the reasoning that follows.

The findings consequently suggest that conjecturing is not merely an initial step that precedes mathematical procedures. It helps determine how students understand the problem and what mathematical actions they consider appropriate. When the initial interpretation is incomplete or inaccurate, subsequent manipulation and the conclusion reached may be affected as well. For non-routine fraction problems, therefore, students need to make sense of the given situation before committing to a particular mathematical procedure. The quality of this initial reasoning can influence the coherence of the solution developed in the later stages.

Mathematical Manipulation in Solving Non-Routine Fraction Word Problems

Most students showed evidence of mathematical manipulation when working on the non-routine fraction word problems. However, conducting the required operations did not always lead to a correct solution. In several responses, students were able to perform the computational steps in a relatively systematic way, but the mathematical model established at the beginning of the solution was inappropriate because the mixed fraction in the problem had been interpreted incorrectly. The written responses, together with the interview data, suggest that the difficulty was therefore not limited to conducting the operation itself. Rather, it was related to how students understood the fractional quantity before deciding what mathematical operation to use. An inaccurate conceptual interpretation could consequently lead to an inappropriate operation, even when the subsequent procedure was performed correctly.

This pattern highlights the close relationship between conceptual understanding and procedural performance in fraction problem solving. Conceptual understanding allows students to recognize mathematical relationships and use them appropriately when solving problems (Copur-Gencturk, 2021). This is particularly relevant in contextual fraction tasks, where knowing how to perform a procedure does not necessarily ensure that students can use it appropriately (Novita et al., 2025). Students also need to understand relationships among fractional quantities rather than approach fraction operations as isolated symbolic rules (Bruce et al., 2023). Different representations can support this process by helping students interpret the quantities involved before applying formal operations, as indicated by the systematic review of Mohamed et al. (2021). Understanding fraction magnitude is likewise important because it can influence the strategies students choose and help them avoid misconceptions when reasoning about fractions (D'Erchie et al., 2025). These perspectives help explain the pattern observed in the present study: procedural performance is more useful when the mathematical procedure is grounded in an appropriate conceptual interpretation.

The findings therefore point to mathematical manipulation as an interaction between knowing how to perform a procedure and understanding why that procedure is appropriate for the situation. In the present study, students could conduct calculations systematically, but an incorrect initial representation sometimes prevented those calculations from producing a valid solution. Fraction instruction should therefore provide opportunities for students to connect fractional quantities with their representations and corresponding procedures, particularly when they need to determine an appropriate operation in non-routine problems.

Drawing Logical Conclusions in Solving Non-Routine Fraction Word Problems

Drawing logical conclusions was the indicator that appeared most frequently across the students' responses. This frequency, however, should not be taken to mean that the conclusions were always supported by sound reasoning. In several cases, the written responses and interview data showed that students' conclusions were consistent with the procedures they had conducted, even though those procedures had originated from an inaccurate interpretation of the problem. Thus, the form of a final answer alone provides limited information about the quality of students' reasoning. To understand the conclusion more fully, it is necessary to consider how the student interpreted the problem, selected and conducted the mathematical procedures, and connected the resulting work to the answer.

This interpretation is supported by several perspectives on mathematical reasoning. Hendriana et al. (2018) argued that mathematical reasoning involves more than obtaining a correct answer, since students also need to construct mathematical arguments that support their conclusions. Shahbari & Peled (2017) similarly emphasized the connection between the information presented in a problem, the procedures used to address it, and the justification offered for the resulting conclusion. The importance of maintaining these connections is also evident in Nusantara et al. (2025), who described a convincing mathematical argument as one in which evidence, reasoning, and justification remain logically related throughout the solution. Brodie (2009) viewed logical conclusions as part of an integrated reasoning process in which the preceding stages contribute to the final outcome. This understanding is consistent with the systematic review by Hjelte et al. (2020), which characterized mathematical reasoning as involving interconnected forms of reasoning rather than isolated skills. Nurazizah & Zulkardi (2022) likewise found that examining the coherence of students' reasoning provides a more informative picture of their mathematical reasoning than considering the correctness of the final answer alone.

In light of these findings, drawing a logical conclusion in a non-routine fraction problem is better understood as the outcome of the reasoning that precedes it. The conclusion reflects, at least in part, how students have interpreted the problem and the mathematical work they have subsequently undertaken. Consequently, an error introduced during conjecturing or mathematical manipulation may remain evident in the final conclusion, even when the student is able to express the answer clearly. This pattern reinforces the view that students' mathematical reasoning should be examined as a connected process, with the quality of reasoning at one stage influencing what is possible at the stages that follow.

Theoretical Implications

The three reasoning indicators examined in this study appear to form a connected pattern in students' problem solving rather than functioning as independent abilities. Difficulties observed when students formed an initial conjecture could influence the mathematical procedures they selected and conducted, and these difficulties could subsequently be reflected in the conclusions they produced. The results therefore suggest that students' reasoning needs to be considered across the sequence of their solution process, since the quality of reasoning at one point may affect what follows.

This interpretation is consistent with broader accounts of mathematical reasoning. In their systematic review, Hjelte et al. (2020) described reasoning as a multidimensional activity in which different forms of reasoning interact rather than operate as isolated skills. Brodie (2009) likewise portrayed mathematical reasoning as an integrated process that includes making sense of a problem, engaging in mathematical work, and providing justification. From this perspective, Nurazizah & Zulkardi (2022) argued that the coherence of students' reasoning provides a more informative indication of mathematical reasoning than the correctness of the final answer considered separately.

The findings also provide a more detailed perspective on the relationship between conceptual understanding and procedural knowledge in fraction learning. Novita et al. (2025) emphasized the importance of integrating these forms of knowledge, while the present findings show how an inaccurate conceptual interpretation can influence the mathematical procedures students choose and, subsequently, the conclusions they formulate when solving non-routine problems. The results also complement the work of Anggraini et al. (2023) by showing more closely how mathematical reasoning is expressed as elementary school students work through contextual fraction problems, rather than considering reasoning only in terms of students' performance on particular indicators.

Overall, the evidence from this study supports viewing mathematical reasoning in non-routine fraction problem solving as a continuous and interconnected process. Students' initial interpretation and conjecture can shape the mathematical manipulation that follows, which in turn can influence the conclusion eventually reached. This interpretation provides a basis for further investigation of students' reasoning processes in elementary mathematics education, particularly studies concerned with how learning experiences can support the development of reasoning across these connected stages.

Practical Implications

The results have several implications for the teaching of fractions in elementary classrooms. Attention to correct answers and procedural accuracy needs to be accompanied by opportunities for students to make their reasoning visible while working on fraction problems. Rather than introducing a procedure at the outset, teachers can ask students to identify relevant information, propose an initial approach, explain the mathematical operations they choose, and support their conclusions with appropriate evidence. Such classroom practices may help students connect their understanding of fractions with the decisions they make when facing problems that are less familiar.

This orientation is consistent with PMRI, which places meaningful contexts and classroom interaction at the center of students' construction of mathematical meaning and reasoning (Putri & Zulkardi, 2019). For fraction learning, this also means drawing students' attention to relationships among quantities instead of treating operations as isolated symbolic rules (Bruce et al., 2023). Multiple representations can help make these relationships accessible and allow students to connect various aspects of fraction concepts (Mohamed et al., 2021). Fraction models and manipulatives provide another means of linking abstract ideas with representations that students can interpret more readily (Wilkie & Roche, 2023). In addition, instructional designs that give students opportunities to express and communicate their mathematical thinking can support reasoning during problem solving (Hidayat et al., 2022; Nabila et al., 2024). Formative feedback can further extend these opportunities by prompting students to explain the reasoning behind their responses and justify the conclusions they formulate (Smit et al., 2023).

These considerations indicate that non-routine fraction word problems can serve purposes beyond assessing whether students obtain the expected answer. When used as part of classroom learning, such problems can provide opportunities for students to interpret information, formulate conjectures, work with mathematical representations and procedures, explain their reasoning, and draw conclusions. Students' responses can consequently provide teachers with information not only about whether a solution is correct, but also about how students are reasoning and which aspects of the process may require further instructional attention.

CONCLUSION

This study examined the mathematical reasoning demonstrated by elementary school students while solving non-routine word problems involving fractions. The results showed differences in how the three reasoning indicators appeared in students' work. Making conjectures emerged as the most demanding aspect, particularly when students had to identify relevant information and establish an initial idea for approaching the problem. Drawing logical conclusions, on the other hand, was observed more frequently, although a conclusion was not necessarily supported by sound reasoning in the preceding stages. Overall, the findings suggest that students' mathematical reasoning is best viewed as a sequence of related activities rather than as separate skills. Students need to interpret the information in a problem and establish an initial conjecture before engaging in mathematical manipulation and eventually formulating a conclusion. An inappropriate interpretation at the outset may influence the mathematical work that follows and, consequently, the conclusion reached by the student. Because the study involved only six students from a single elementary school, the findings are closely tied to the participants and setting examined and should not be generalized to elementary students in other contexts without further evidence. Future studies could examine a wider range of students and settings and investigate instructional approaches that help students strengthen the connections among the various stages of reasoning when solving non-routine fraction word problems.

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