



How AI Facilitates Students in Constructing Ideas to Solve Mathematical Problems: A Critical Thinking Perspective

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Abstract

Critical thinking is an essential skill in the 21st century; however, many students continue demonstrating low critical thinking abilities, partly attributed to the lack of personalized learning approaches. This lack of learning approaches leads students to look for other alternatives to comprehend subjects at school better, and one of the ways is by utilizing AI. One real example of students utilizing AI happens in Mathematics class, and this phenomenon stole the researchers' interest to conduct further study. This study aimed to investigate how AI, mainly GenAI platforms (i.e., Meta AI and ChatGPT), facilitates the construction of mathematical ideas in comprehending problems and formulating appropriate solution strategies. The research was conducted at a school in Surabaya during the second semester of the 2024/2025 academic year, involving five students aged 15-17 years who are basically capable of utilizing AI on a daily basis without prior training in learning contexts. A qualitative research design within an interpretative paradigm was employed, and the data analysis was conducted through Reflexive Thematic Analysis to identify, analyze, and report patterns within the data. The findings indicate that AI facilitates the construction of mathematical ideas based on critical thinking indicators through mechanisms of cognitive scaffolding, iteration and refinement, and prompt development following a general-to-specific pattern. The construction of ideas that occurred during the research take into account students' initial conditions, thereby demonstrating AI's role in enhancing personalized mathematics learning.

Keywords: artificial intelligence; critical thinking; constructing ideas



INTRODUCTION

Critical thinking is one of the essential tools used to solve problems, both in everyday life and in educational contexts. Critical thinking involves the ability to interpret, analyze, evaluate, and draw conclusions or inferences. According toutama et al. (2022), learners are said to have good critical thinking skills when they can meet the indicators of critical thinking, namely solving problems with simple solutions, drawing conclusions accompanied by logical reasons, elaborating answers, as well as planning and formulating strategies to determine the best outcome. In the context of mathematics learning, in addition to being a tool for solving problems logically, critical thinking skills can also be used as a reflective process for learners to gain a meta-perspective on their own mathematics teaching and learning process (Sachdeva & Eggen, 2021).

The development of AI in the 21st century plays a significant role in mathematics education; AI inference has great potential to enhance pedagogy and manage cognitive load (Patac & Patac, 2025). Artificial Intelligence can support more efficient learning by simplifying complex mathematical concepts, while still providing structured explanations, thus allowing the learning process to be proceeded more quickly. Machines or AI assist learners in developing thinking skills by providing information related to specific topics, highlighting unexplored aspects, and introducing learners to unexplored areas within that topic (Almarashdi et al., 2024). AI integrated with learning serves as a catalyst for interactive and feedback-based learning; it can be said that AI is an effective tool for enhancing critical thinking skills (Panit, 2024).

Nevertheless, AI also has limitations, one of which is in handling complex mathematical problems, such as topics geometry (Udias et al., 2024). The feedback provided by AI in understanding word-based mathematical questions depends on the prompts given and the data entered (Egara & Mosimege, 2024). Continuous use of AI can lead to dependency and affect the development of learners' critical thinking skills (Panit, 2024; Sánchez-Ruiz et al., 2023).

Research conducted by Soesanto et al. (2022) explains that AI offers the use of ITS or Intelligent Tutoring System models in mathematics learning, with a list of mathematics topics originating from students that have the potential to be developed with AI. The integration of AI in learning can help students solve mathematical problems, including those complex ones and code creation related to computational thinking. In the context of mathematics learning, AI also has the potential to develop students' skills to think critically; for example, in the construction of ideas for mathematical proofs, artificial intelligence can generate mathematical arguments to encourage students to compare and validate these additional sources, thereby providing students with many opportunities to understand the fundamental nature of proof and foster confidence to substantiate independently (Yoon et al., 2024).

However, many educators fear the long-term impact of AI on students' critical thinking skills. This concern arises due to the risk of reliance on technology, which could reduce the initiative for independent thought. It is important to have awareness and recognize limitations when using AI. Fundamentally, AI relies on algorithms that utilize statistical patterns and probabilities from training data, making it possible for solutions generated by AI to be incorrect, nonsensical, or incoherent (Gouia-Zarrad & Gunn, 2024). Therefore, the researchers decided to put their focus on conducting this research on how the use of AI, particularly GenAI, in the construction of mathematical ideas can improve students' critical thinking skills.

This research integrates three main concepts, namely critical thinking, idea construction, and AI facilitation, using critical thinking indicators (Facione, 1990; Sachdeva & Eggen, 2021) with four phases of mathematical idea construction to explain how AI can facilitate students' mathematical thinking processes (Chan & Kwan, 2021). This research aims to determine how AI plays a role in facilitating mathematical idea construction, especially in understanding given problems and formulating appropriate solution strategies, thereby explaining the impact of AI use on the development of students' critical thinking skills in the context of problem-solving. Accordingly, this study addresses two research questions: how AI helps students formulate ideas for solving mathematical problems, and how AI influences the improvement of students' critical thinking skills.

METHOD

Research Design

The research design used is a qualitative research design within an interpretative paradigm. Qualitative data was analyzed using Reflexive Thematic Analysis (Braun & Clarke, 2006; Ahmed et al., 2025), which involved the process of identifying, analyzing, and reporting patterns (themes) within the data. This method has been proven effective in mathematics education research regarding the integration of generative AI technology, particularly in analyzing teachers' reflections and interactions with AI (Labis, 2025).

Participants

This research was conducted at a school in Surabaya in the second semester of the 2024/2025 academic year. The study subjects comprised five individuals aged 15-17 years, representing students capable of using AI, but without prior training in its use for learning. The purpose of selecting participants with these criteria was to ensure a diversity of perspectives when providing insights into reflective thinking, both during tasks and after using GenAI tools in learning.

Instruments Used and Data Analysis

The data analyzed consisted of test answers before using AI and test answers after using AI, interview results, as well as prompts and screen recordings when using AI. Then, the initial prompts entered or given to the AI as well as the results obtained from those prompts were also examined. After that, the results were compared to the steps in the test answers before using AI. Finally, any iterations that participants performed were counted furthermore to achieve their desired final results.

The kind of AI that was utilized by participants in this research is GenAI, and it was accessed via free accounts on web and chat applications. The specific AI tools that were used mirrored those employed by the participants. Based on the data obtained, many participants used ChatGPT and Meta AI as they were considered easier to use and their explanations were easier to understand.

In this study, the qualitative data obtained were analyzed using Reflexive Thematic Analysis (RTA) (Braun & Clarke, 2006; Ahmed et al., 2025), which is consistent with the interpretivist paradigm and the exploration of students' reflective experiences in using GenAI tools. To guide the analysis systematically, this study employed an analytical framework combining two theoretical constructs: the mathematical idea construction phase and critical thinking indicators.

In the analysis process, the mathematical idea construction phase was used as an initial coding framework or a priori code. Each cognitive stage that appeared in the data was coded according to the most relevant construction phase, as detailed in Table 1 below.

Table 1. Phase1 of Mathematical Idea Construction

Phase/Stage	Description
Translation	Creating a representation in the form of an image
Integration	Transforming relevant information into an integrated mathematical representation, which can help in designing strategies to solve problems
Planning	Designing possible strategies and trying various methods through trial and error
Execution	Executing the plan, which involves calculations.

Meanwhile, critical thinking indicators (Facione, 1990; Sachdeva & Eggen, 2021) served as interpretive criteria for assessing and understanding the quality of students' cognitive processes. The six dimensions allow researchers to evaluate how deeply students think at each stage, as presented in Figure 1.

Interpretation		
Categorization	Decoding Significance	Clarifying Meaning
Analysis		
Examining Ideas	Identifying Arguments	Analyzing Arguments
Evaluation		
Assessing Claims		Assessing Arguments
Inference		
Querying Evidence	Conjecturing Alternatives	Drawing Conclusions
Explanation		
Stating Results	Justifying Procedures	Presenting Arguments
Self-regulation		
Self-examination		Self-correction

Figure 1. Indicators of Critical Thinking

The analytical process was carried out through six systematic phases of RTA with an integrated framework applied in each phase. The first phase, familiarizing with the data, began with repeatedly reading interview transcripts, individual student reflections, and their work outputs to gain a deep understanding and identify initial patterns. In the second phase, generating initial codes, the researchers performed both inductive and inductive coding where each meaningful data segment was assigned a descriptive code label. Entering the third phase, searching for themes, the generated codes were grouped into broader patterns or themes. In this phase, the researchers began to note down potential themes and collect relevant data excerpts for each theme. The fourth phase, reviewing themes, involve reviewing the themes to ensure internal coherence within each theme and clearly differentiate between themes. The researchers returned to the raw data to verify that the formulated themes accurately represented the data. In the fifth phase, defining and naming themes, the researchers refined the essence of each theme by providing clear definitions and descriptive names, and explaining how these themes relate to the research questions. In the final phase, writing the report, the researchers presented the findings through a coherent narrative by integrating data excerpts to illustrate each theme and explicitly link them to the research questions.

RESULTS

Critical thinking skills are a fundamental asset for adapting to the 21st century (Setiana et al., 2021). Through critical thinking, students can develop analytical and systematic thinking that can improve understanding in various aspects of learning. Güner and Erbay (2021) state that mathematics learning involves several skills, including problem-solving techniques, problem identification, and formulating statements from incomplete information. However, Setiana et al. (2021) and Utami and Pramudiani (2024) noted that Indonesian students' mathematical critical thinking skills are still relatively low and impact their understanding of subjects, particularly mathematics. This statement is supported by the results of the 2015 TIMSS, which ranked Indonesia 45th out of 50 participating countries. In

response to this problem, various studies have developed alternative learning methods, one of which is the integration of AI. The researchers conducted a study related to the integration of AI in mathematics learning to improve students' critical thinking abilities.

Data analysis yielded five distinct findings from five research subjects, each demonstrating varying critical thinking skills and interaction patterns with AI. Table 2 summarizes the themes, codes, and quotes from participants while working on the questions before and after using AI.

Table 2. Theme, Codes, and Example Participant Quote

Themes	Codes	Examples of Participants' Quotes Without AI	Examples of Participants' Quotes With AI
Interpretation	Categorization, Unpacking Meaning, Clarifying Meaning	"...So, you see, because there are three of these, I initially wanted to use two first. The thing is, if there are three, I'd be confused, you know, the small one and the biggest one..." – participant 2 (Ethak)	"...Its description in those words, and consistent with my thoughts. So, for example, there are two formulas..." – participant 1 (Lisa)
Analysis	Examining Ideas, Identifying Arguments, Analyzing Arguments	"...I'm looking at this shape, and it looks like it's a right-angled triangle. But I'm not sure. Then I tried it. I tried a right-angled triangle, because its side lengths were already known from the formula for externally tangent circles..." – participant 2 (Ethak)	"...There are two. Both use the property of tangent circles. But, it seems that the first characteristic is wrong. The second one also utilizes the property of tangent circles. But the difference is it forms in the roots,..." – participant 2 (Ethak)
Evaluation	Assessing Claims, Assessing Arguments	"...If thought about logically and based on the actual illustration, my answer isn't correct, because here it's known that the radius of the medium circle is only 1 cm, whereas my answer for the radius of the small circle is 1.5 cm, so it's likely wrong..." – Participant 3 (Raka)	"...I think this answer is correct because it makes more sense than my previous answer. Because the previous one was 1.5 cm, or larger than the radius of the medium circle. However, the answer given by the AI found the radius of the small circle to be $\frac{4}{9}$ cm, and if that's converted to a decimal, it should be below 1..." – Participant 3 (Raka)
Inference	Questioning Evidence, Considering Other Alternatives, Drawing Conclusions	"...Initially, I wanted to draw them touching, but it was difficult, so I tried another drawing and it turned out I got the tangent line..." – Participant 1 (Lisa)	"...Of the two methods given by the AI, I chose the one using Pythagoras' theorem because there was a triangle diagram and I didn't know where the other formula came from..." – Participant 1 (Lisa)

Themes	Codes	Examples of Participants' Quotes Without AI	Examples of Participants' Quotes With AI
Explanation	Stating Results, Justifying Procedures, Presenting Arguments	"...Draw it first. Sometimes, I interpret the drawing as the small circle being squeezed in. Like what I tried earlier with the parallel lines, there was an indication that this could use a tangent line..." – Participant 1 (Lisa)	"...From the two trials earlier, the answers were different, and that was very clear from the size of the circles. Because the smallest circle was being hunted, it was clearly going to be smaller than the medium circle. So, the result must be less than 1 cm..." – participant 2 (Ethak)
Self-Regulation	Self-Monitoring, Self-Correction	"...Then B squared is BC, which is unknown. Then after that, I calculated it using Pythagoras. But, it turns out that so far I haven't found what x is. Instead, I've just been finding y-values all along..." – participant 2 (Ethak)	"...Besides, I saw that all the answers were like mine too. I looked closely at the triangle, and I think it makes sense. So, from the explanation, it's like I just understood it..." – participant 1 (Lisa)

AI Facilitating the Fulfillment of Interpretation and Analysis Indicators

Participant 1's (Lisa's) Work Without AI

Based on the results of the analysis of Lisa's work in Figure 2, it is known that her interpretation, analysis, and evaluation abilities were met. Lisa could state the known information in the problem using a 'known' and 'asked' format and visualize the problem in the question with three circle diagrams

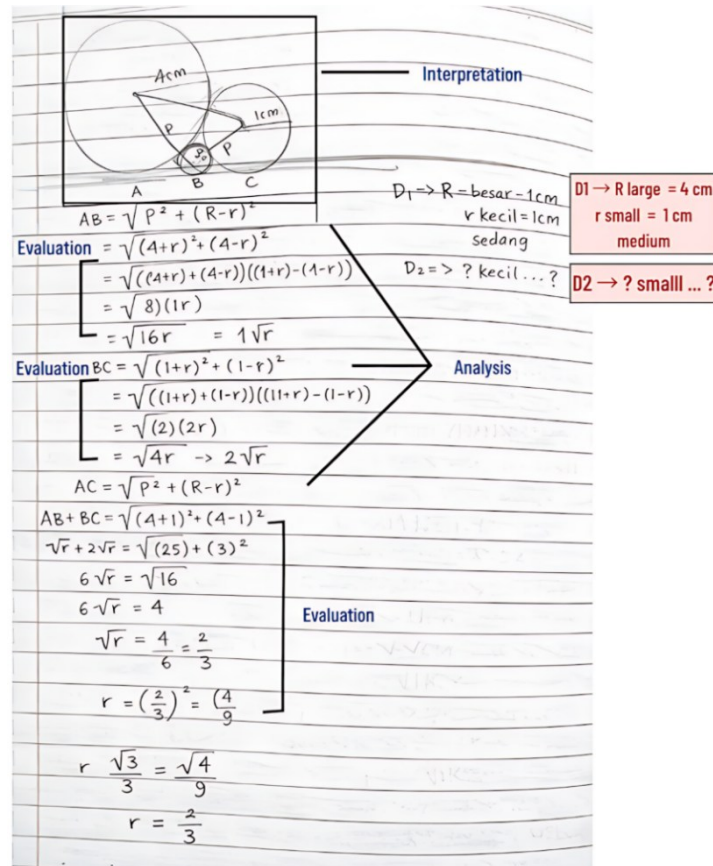


Figure 2. Lisa's Work without Using AI

This statement is supported by the following interview results (Note: T = Teacher, S = Student)

- T : So, what information did you get from this problem?
 S : Maybe, the radius of the big circle, the radius of the small circle, and also that even the smaller circle is kind of squeezed between the two circles.
 T : Do you have any idea what the circles and their arrangement look like?
 S : Initially, I wanted to draw them touching, but it was difficult. So, I tried drawing something else and found the tangent line.
 T : How did you know the circles should look like this?
 S : Actually, at first, I tried arranging the small circles, so they were aligned in one straight line, with this one in the middle, but I couldn't find the answer. So, I just tried moving it downwards.

From the interview transcript and the image, it is evident that Lisa experimented in problem-solving, such as trying to draw circles in different positions to find an easier solution.

Participant 1's (Lisa's) Work Using AI

When using AI, there was a reinforcement of the idea construction process during the interpretation and analysis stages. Based on Figure 3, AI provided clarity/confirmation regarding Lisa's interpretation.

card lain: $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + 2\sqrt{\frac{1}{r_1 r_2}}$ $R = k e c i t, r_1 = b e s a r, r_2 = k e c i t$

Analysis

$$\frac{1}{R} = \frac{1}{4} + 1 + 2\sqrt{\frac{1}{4 \cdot 1}}$$

$$\frac{1}{R} = \frac{1}{4} + 1 + 2\sqrt{\frac{1}{4}}$$

$$\frac{1}{R} = \frac{1}{4} + 1 + 2 \cdot \left(\frac{1}{2}\right)$$

Evaluation

$$\frac{1}{R} = \frac{1}{4} + 1 + 1$$

$$\frac{1}{R} = \frac{1}{4} + 2$$

$$\frac{1}{R} = \frac{9}{4}$$

$$R = \frac{4}{9}$$

other method: R = small;
r1 = large; r2 small

Figure 3. Lisa's Work with Using AI

This statement is supported by the following interview results. (Note: T = Teacher, S = Student)

- T : Did the AI provide an illustration or visualization of the problem?
- S : The illustration was in the form of words, and it matched my thoughts.
- T : Did the AI provide information to solve the problem?
- S : Yes. It defined two formulas, but I only used this one. It defined the 'R' (capital R) as the radius of the small circle being searched for. Then, R1 is the radius of the large circle, and R2 is the radius of the medium circle. It gave two methods earlier, one using Pythagoras, and the other one, which I actually asked AI where it came from, couldn't be explained.
- T : So, then you chose Pythagoras. Do you think it's suitable for solving the problem?
- S : I think so because the result was the same. Besides that, there was a triangle diagram involved.
- T : Of the two methods given, which one do you find clearer?
- S : The Pythagorean method, and the reason is because I don't know where the other method came from. Meanwhile, this one clearly used the Pythagorean formula
- Lisa performed 3 iterations to ensure the information provided by the AI was accurate.



Meta AI
with Llama 4

1. Misalkan kita memiliki tiga lingkaran yang saling bersinggungan, yaitu lingkaran besar dengan jari-jari R_1 , lingkaran sedang dengan jari-jari R_2 , dan lingkaran kecil dengan jari-jari r .

2. Buatlah garis yang menghubungkan pusat-pusat ketiga lingkaran.

3. Dengan menggunakan teorema Pythagoras, kita dapat menentukan hubungan antara jari-jari lingkaran besar, lingkaran sedang, dan lingkaran kecil.

Rumus yang dihasilkan adalah:

$$1/r = 1/R_1 + 1/R_2 + 2 * \sqrt{1/(R_1 * R_2)}$$

Rumus ini dapat digunakan untuk menentukan jari-jari lingkaran kecil yang terhimpit antara dua lingkaran yang saling bersinggungan.

Referensi

Rumus ini dapat ditemukan dalam berbagai sumber, termasuk buku-buku geometri dan artikel ilmiah. Salah satu sumber yang populer adalah "Descartes' Theorem" atau "Kissing Circles Theorem", yang menjelaskan tentang hubungan antara lingkaran yang saling bersinggungan.

Dengan menggunakan rumus ini, kita dapat menentukan jari-jari lingkaran kecil yang terhimpit antara dua lingkaran yang saling bersinggungan dengan lebih mudah dan akurat.

3:27 PM

1. Suppose we have three mutually tangent circles, namely a large circle with radius R_1 , a medium circle with radius R_2 , and a small circle with radius r .

2. Draw lines connecting the centers of the three circles.

3. By applying the Pythagorean theorem, we can determine the relationship among the radii of the large circle, medium circle, and small circle.

The resulting formula is:

$$1/r = 1/R_1 + 1/R_2 + 2 * \sqrt{1/(R_1 * R_2)}$$

This formula can be utilized to determine the radius of the small circle that is tangent to two mutually tangent circles.

References

This formula can be found in various sources, including geometry textbooks and scholarly articles. One popular source is "Descartes' Theorem" or "Kissing Circles Theorem," which explains the relationship among mutually tangent circles. By employing this formula, we can determine the radius of the small circle that is tangent to two mutually tangent circles with greater ease and accuracy..

jelaskan lebih rinci 3:29 PM ✓✓

Please explain in more detail

Rumus yang saya gunakan untuk menentukan jari-jari lingkaran kecil yang terhimpit antara dua lingkaran yang saling bersinggungan adalah berdasarkan pada teori geometri lingkaran dan teorema Descartes.

The formula I utilized to determine the radius of the small circle that is tangent to two mutually tangent circles is based on circle geometry theory and Descartes' theorem.

Figure 4. Lisa's 1 Iteration

After entering an initial prompt in the form of a question, the AI provided a direct solution.

From the Figure 4, it can be seen that, after the AI provided a method for solving the problem using a specific formula, Lisa performed the first iteration by asking the AI to explain the given answer in more detail.



Meta AI
with Llama 4

$$r = (4 \times 1) / (4 + 2 \times \sqrt{4 \times 1} + 1)$$

$$r = 4 / (4 + 2 \times 2 + 1)$$

$$r = 4 / (4 + 4 + 1)$$

$$r = 4 / 9$$

Jari-jari lingkaran kecil adalah 4/9 cm.

Namun, ada cara lain yang lebih sederhana untuk menyelesaikan masalah ini dengan menggunakan rumus:

$$1/R = 1/r_1 + 1/r_2 + 2 \times \sqrt{1/(r_1 \times r_2)}$$

dimana R adalah jari-jari lingkaran kecil, r1 adalah jari-jari lingkaran besar, dan r2 adalah jari-jari lingkaran sedang.

Dengan menggunakan rumus ini, kita dapat menentukan jari-jari lingkaran kecil dengan lebih mudah.

$$1/R = 1/4 + 1/1 + 2 \times \sqrt{1/(4 \times 1)}$$

$$1/R = 1/4 + 1 + 2 \times \sqrt{1/4}$$

$$1/R = 1/4 + 1 + 2 \times 1/2$$

$$1/R = 1/4 + 1 + 1$$

$$1/R = 1/4 + 2$$

$$1/R = 9/4$$

$$R = 4/9$$

Jadi, jari-jari lingkaran kecil adalah 4/9 cm.

3:22 PM

$$r = (4 \times 1) / (4 + 2 \times \sqrt{4 \times 1} + 1)$$

$$r = 4 / (4 + 2 \times 2 + 1)$$

$$r = 4 / (4 + 4 + 1)$$

$$r = 4 / 9$$

The radius of the small circle is 4/9 cm.

However, there is a more simplified method to resolve this problem using the following formula:

$$1/R = 1/r_1 + 1/r_2 + 2 \times \sqrt{1/(r_1 \times r_2)}$$

where R represents the radius of the small circle, r1 denotes the radius of the large circle, and r2 signifies the radius of the medium-sized circle.

By employing this formula, we can determine the radius of the small circle with greater efficiency:

$$1/R = 1/4 + 1/1 + 2 \times \sqrt{1/(4 \times 1)}$$

$$1/R = 1/4 + 1 + 2 \times \sqrt{1/4}$$

$$1/R = 1/4 + 1 + 2 \times 1/2$$

$$1/R = 1/4 + 1 + 1$$

$$1/R = 1/4 + 2$$

$$1/R = 9/4$$

$$R = 4/9$$

Consequently, the radius of the small circle is 4/9 cm.

darimana rumus yang kamu pakai itu berasal

3:27 PM ✓

What is the derivation or origin of the formula you utilized?

Rumus yang saya gunakan untuk menentukan jari-jari lingkaran kecil yang terhimpit antara dua lingkaran yang saling bersinggungan berasal dari teori geometri lingkaran.

Penurunan Rumus

Rumus tersebut dapat diturunkan dengan menggunakan konsep geometri lingkaran dan teorema Pythagoras. Berikut adalah langkah-langkah penurunan rumus:

The formula utilized to determine the radius of a small circle inscribed between two tangent circles is derived from the principles of circle geometry.

Formula Derivation

The aforementioned formula can be derived by applying the concepts of circle geometry and the Pythagorean theorem. The following are the steps involved in the derivation:

Figure 5. Lisa's 2 Iteration

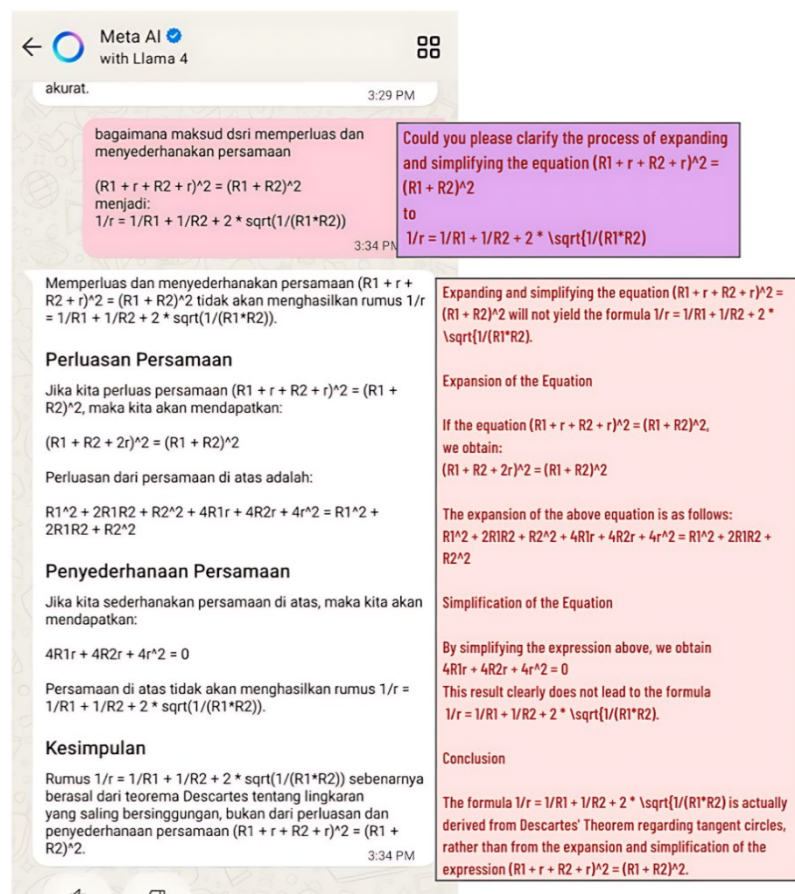


Figure 6. Lisa's 3 Iteration

In Figures 5 and 6, Lisa performed repeated iterations to ensure the information provided by the AI was valid. These iterations demonstrate a critical attitude towards the AI's answer, where Lisa did not immediately accept the answer but also performed verification.

Participant 2's (Ethak's) Work Without AI

Based on the analysis of Ethak's work in the Figure 7, it is known that interpretation and analysis skills were present, but Ethak could not find the final result for the given problem

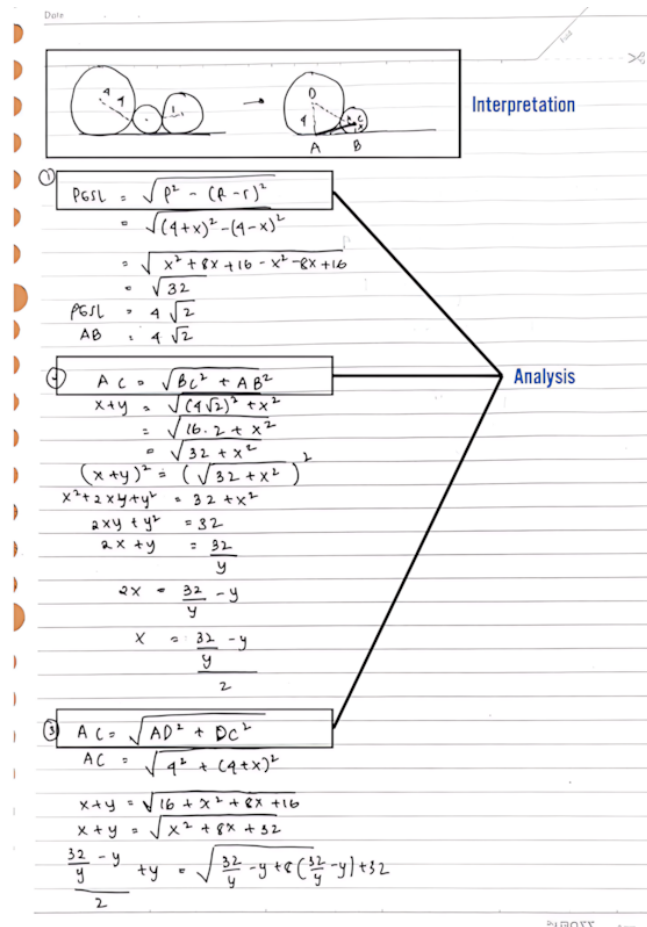


Figure 7. Ethak's Work without Using AI

Ethak's interview results are supported by the number of iterations performed. At this stage, an interaction occurred between Ethak and the AI because the AI provided two formulas that could be used in the solution.

This statement is supported by the following interview results (Note: T = Teacher, S = Student)

- T : What information did you get from that problem?
- S : There were three circles, such as a large one, a medium one, and a small one. Those were tangent to each other. The large one had a radius of four, the medium one had a radius of one, and the radius of the small one was still being looked for.
- T : Do you have any shape or visualization of the circles?
- S : Yes. There were three circles. I initially wanted to use two first. Because if I used three, which consisted of the small and the largest ones, I got confused. Then, because we didn't know where the tangency point was, I placed them separately at the bottom, so they were aligned. Then, because they were tangent, I drew this here, the radii. Well, this radius is four, and this one is unknown.
- T : Do you think the method you're using is suitable for solving this problem?
- S : I don't think so. Because, instead of finding the radius, I kept going in circles calculating lines that I probably shouldn't do.

In her work, Ethak had already experimented with using appropriate formulas and interpreting the image, but she had not yet found a solution to the problem.

Participant 2's (Ethak's) Work Using AI

Based on the analysis of Ethak's work using AI assistance, Ethak was able to find the answer to the problem in the question. Figure 8 shows the short solution provided by AI.

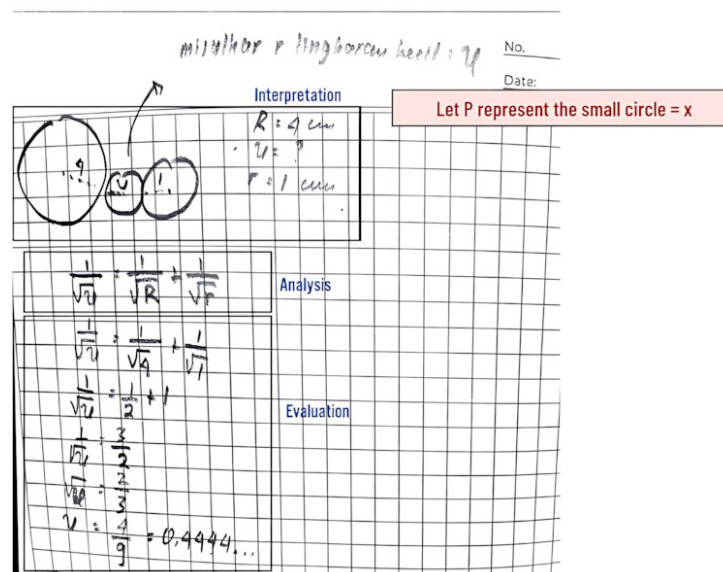


Figure 8. Ethak's Work with Using AI

This statement is supported by the following interview results. (Note: T = Teacher, S = Student)

- T : Did the AI provide any visualizations or images earlier?
 S : No, it immediately gave formulas. It gave two attempts, but the first attempt was wrong.
 T : How did you ensure that the answer provided by the AI was correct?
 S : From the two attempts, the answers were different, and it was very obvious from the size of the circles. Since the smallest circle was being searched for, it was clearly going to be smaller than the medium circle. So, the result must be less than 1 cm.

These interview results are also supported by the number of iterations that he performed. Ethak carried out 7-8 iterations to ensure that the answer provided by the AI was valid.

Diketahui tiga buah lingkaran saling bersinggungan berukuran besar, sedang, dan kecil. Lingkaran besar memiliki jari-jari sebesar 4 cm, lingkaran sedang memiliki jari-jari sebesar 1 cm, dan lingkaran kecil terhimpit antara lingkaran besar dan lingkaran sedang. Berapa jari-jari lingkaran kecil? (Gunakan format Diketahui, Ditanya, dan Dijawab) 4:09 PM ✓

There are three mutually tangent circles. The radius of the large circle is 4 cm, and the radius of the medium circle is 1 cm. Determine the radius of the smallest circle!

Diketahui

- Jari-jari lingkaran besar (R) = 4 cm
- Jari-jari lingkaran sedang (r) = 1 cm
- Lingkaran kecil terhimpit antara lingkaran besar dan lingkaran sedang

Ditanya

- Jari-jari lingkaran kecil (x)

Dijawab

Misalkan jari-jari lingkaran kecil adalah x. Berdasarkan sifat lingkaran yang bersinggungan, kita dapat membuat persamaan berikut:

Known

- Radius of the large circle (R) = 4 cm
- Radius of the medium circle (r) = 1 cm
- The small circle is tangent to both the large and medium circles.

Asked

- The radius of the small circle (x)

Answer

Let the radius of the small circle be denoted by x. Based on the properties of tangent circles, we can establish the following equation:

Figure 9. Ethak's 1 Interaction with AI

After entering the initial prompt in the form of a question, the AI provided an immediate answer. From Figure 9, it can be seen that, after the AI provided a way to solve the problem using a certain formula, Ethak conducted the first iteration by asking the AI to explain in more detail regarding the answer and the solution strategy provided, as shown in Figure 10.

Sifat yg apa 4:10 PM ✓

What specific property is being applied?

Sifat lingkaran yang bersinggungan dan digunakan dalam perhitungan jari-jari lingkaran kecil adalah:

The properties of tangent circles used in calculating the radius of the smaller circle are:

Rumus untuk jari-jari lingkaran kecil yang terhimpit antara dua lingkaran besar dan sedang yang bersinggungan adalah:

The formula for the radius of the small circle inscribed between two tangent large and medium circles is

$$1/\sqrt{x} = 1/\sqrt{R} + 1/\sqrt{r}$$

$$1/\sqrt{x} = 1/\sqrt{R} + 1/\sqrt{r}$$

where x is the radius of the small circle, R is the radius of the large circle, and r is the radius of the medium circle.

di mana x adalah jari-jari lingkaran kecil, R adalah jari-jari lingkaran besar, dan r adalah jari-jari lingkaran sedang.

This property is based on the concept of tangent circles and can be used to determine the radius of the smaller circle nestled between two larger circles.

Sifat ini berdasarkan pada konsep geometri lingkaran yang bersinggungan dan dapat digunakan untuk menentukan jari-jari lingkaran kecil yang terhimpit antara dua lingkaran lain yang lebih besar.

4:10 PM

Rumusnya pakai akar kok kamu gak kerjain pake akar juga 4:11 PM ✓

Since the formula involves square roots, you should have carried out the calculation using square roots as well.

Anda benar, saya seharusnya menggunakan rumus yang tepat dengan akar untuk menyelesaikan masalah tersebut. Mari kita coba lagi menggunakan rumus yang benar:

You are correct; I should have used the appropriate formula involving square roots to solve the problem. Let us attempt the solution again using the correct formula:

$$1/\sqrt{x} = 1/\sqrt{R} + 1/\sqrt{r}$$

Substitusikan nilai R = 4 dan r = 1:

$$1/\sqrt{x} = 1/\sqrt{4} + 1/\sqrt{1}$$

$$1/\sqrt{x} = 1/2 + 1$$

Substitute the values R = 4R = 4R = 4 and r = 1r = 1r = 1:

Figure 10. Ethak's 2-3 iteration

Ethak iterated because the AI's answer had not helped his understanding of the solution provided. Figure 11 shows that Ethak questioned the reasoning behind the solution strategy used by the AI.

tersebut:

$$\sqrt{x} = 2/3$$

$$x = (2/3)^2$$

$$x = 4/9$$

Jadi, jari-jari lingkaran kecil adalah 4/9 cm. Terima kasih atas koreksinya!

4:11 PM

Meta AI

Diketahui

- Jari-jari lingkaran besar (R) = 4 cm
- Jari-jari lingkaran sedang (r) = 1 cm...

Kenapa kamu bisa bikin persamaan kayak gini

4:12 PM ✓

Why were you able to formulate an equation like this?

Persamaan yang saya buat sebelumnya, yaitu:

$$1/(x+1) + 1/(x+4) = 1/x$$

ternyata tidak tepat untuk menyelesaikan masalah ini.

Persamaan yang benar untuk lingkaran yang bersinggungan adalah:

$$1/\sqrt{x} = 1/\sqrt{R} + 1/\sqrt{r}$$

The equation that I previously formulated was:

$$1/(x+1) + 1/(x+4) = 1/x$$

However, this equation is not appropriate for solving the given problem.

The correct equation for tangent circles is:

Figure 11. Ethak's 4 iteration

Maksudnya kenapa rumusnya kayak gitu asal usulnya darimana kenapa menggunakan akar untuk jari jari

4:20 PM ✓

Rumus $1/\sqrt{x} = 1/\sqrt{R} + 1/\sqrt{r}$ untuk lingkaran yang bersinggungan memiliki asal usul dari konsep geometri dan inversi.

Inversi adalah suatu transformasi geometri yang memetakan lingkaran ke lingkaran lain atau ke garis. Dalam kasus lingkaran yang bersinggungan, inversi dapat digunakan untuk menganalisis hubungan antara jari-jari lingkaran-lingkaran tersebut.

Ketika kita melakukan inversi pada lingkaran yang bersinggungan, kita dapat melihat bahwa jari-jari lingkaran kecil yang terhimpit antara dua lingkaran besar dan sedang memiliki hubungan yang proporsional dengan invers dari jari-jari lingkaran besar dan sedang.

Penggunaan akar dalam rumus tersebut dapat dijelaskan sebagai berikut:

I am asking why the formula takes this particular form, what its origin is, and why square roots are used in relation to the radii.

The formula $1/\sqrt{x} = 1/\sqrt{R} + 1/\sqrt{r}$ for tangent circles originates from concepts in geometry and inversion.

Inversion is a geometric transformation that maps circles to other circles or to lines. In the case of tangent circles, inversion can be employed to analyse the relationship among the radii of the circles.

When an inversion is applied to tangent circles, it can be observed that the radius of the small circle confined between the larger and intermediate circles has a proportional relationship with the inverses of the radii of the larger and intermediate circles.

Figure 12. Ethak's 5 iteration

Ethak's iterations concluded when the AI had provided a convincing answer to Ethak's doubts, as seen in Figure 12.

Participant 3's (Raka's) Work Without AI

Known:
r1 = 4 cm
r2 = 1 cm

Ask:
r small circle?

Answer:

Diketahui:
r1 = 4 cm
r2 = 1 cm

Ditanya:
r lingkaran kecil?

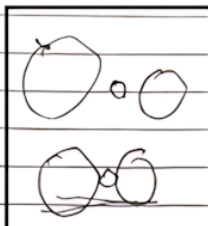
Dijawab:

$$r_3 = \frac{r_1 - r_2}{2}$$

$$r_3 = \frac{4 - 1}{2}$$

$$r_3 = \frac{3}{2}$$

$$r_3 = 1,5 \text{ cm}$$



Interpretation

Analysis

Evaluation

Figure 13. Raka's Work without Using AI

Based on the analysis of Raka's work in Figure 13, it was found that his interpretation, analysis, and evaluation skills were met. Raka could state the information in the problem in a 'given' and 'asked' format and was able to draw a visualization of three circles. However, the solution method used was incorrect and resulted in an inaccurate answer.

This statement is supported by the following interview results. (Note: T = Teacher, S = Student)

- T : What information did you get from the problem?
- S : The information I got from this problem is that there were two circles, and this small circle is squeezed between the large circle and the small circle.
- T : Do you have a shape or visualization based on this problem?
- S : So, there were a large circle and a medium circle. Then, in the middle of them, there was a small circle. The three circles were touching each other.

T : In your opinion, what's the most appropriate way to solve this problem?

S : In my opinion, the most appropriate way would actually be to use a formula with square roots, you know. But, because I haven't covered that material yet, and I still can't do it, I used a simple method that I thought made logical sense.

In his work, Raka had tried several attempts to solve it which was evident from the scribbles of different visual interpretations and interview results. Those stated that Raka finally used another method which he thought was easier than the method that he had previously considered.

Participant 3's (Raka's) Work Using AI

Based on the results of the analysis of work using AI assistance, Raka got the correct answer to the question given. Figure 14 shows that the solution provided by AI is in the form of a root formula, in accordance with what Raka meant in the first interview accompanied by an easy-to-understand explanation.

Figure 14. Raka's Work with Using AI

In his work using AI, Raka did not iterate to ensure the correctness and validity of the answer provided by the AI and immediately accepted the AI's answer without reflecting on its errors.

This statement is supported by the following interview results. (Note: T = Teacher, S = Student)

T : Did the AI visualize the problem?

S : It did. Previously, I had written down the given problem.

T : Based on the information provided by the AI, did the method given by the AI match your method of solving the problem?

S : It's different. The AI provided a formula using roots, as I mentioned before.

T : Do you think there are any errors in the solution provided by the AI, such as the formula perhaps being wrong or the calculation?

S : I haven't asked further questions, but I'm already confident with the answer given by the AI. It is because I think that the answer this time has made more sense than my previous answer.

In Raka's work, there was no evaluation process for the solution provided by the AI. Furthermore, Raka only focused on the correct answer without understanding the thought process that led to that answer, and he felt capable because he "succeeded" with the AI.

Participant 4's (Rafayna's) Work Without AI

The analysis results show that Rafayna did not meet the interpretation, analysis, and evaluation skills because the given problem was not successfully solved, as shown in Figure 15.

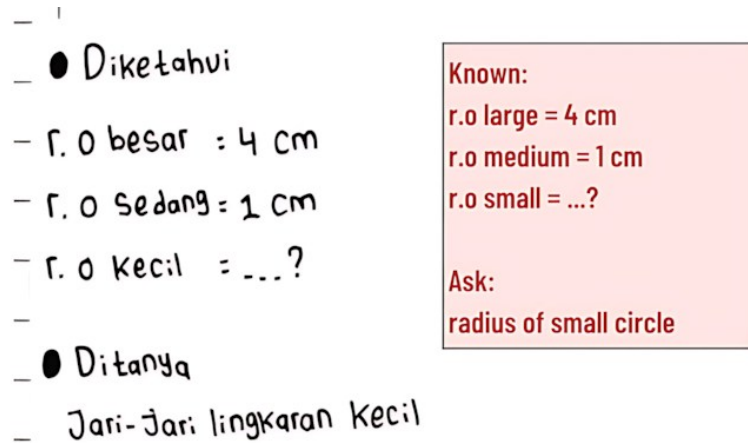


Figure 15. Rafayna's Work without Using AI

This statement is supported by the following interview results (Note: T = Teacher, S = Student)

- T : What information did you get from this question?
S : I was asked to find the radius.
T : What kind of visualization did you think of from that question?
S : The smallest circle, the medium circle, and the large circle were all touching each other.
T : In your opinion, what's the most appropriate way to solve this question?
S : Using a formula, but I don't know which formula, Ma'am.

In her work, Rafayna was unable to identify an appropriate problem-solving strategy.

Participant 4's (Rafayna's) Work Using AI

Based on the analysis of the work carried out with AI assistance, Rafayna obtained the correct answer to the given problem. The solution provided by the AI includes problem interpretation, solution strategy, and the final answer. As shown in Figure 16.

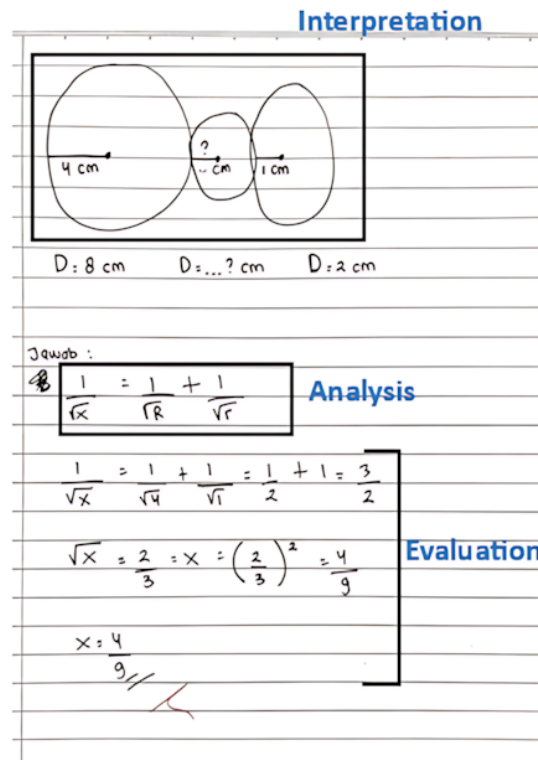


Figure 16. Rafayna's Work with Using AI

This statement is supported by the following interview results (Note: T = Teacher, S = Student)

T : Did Chat GPT provide an illustration or visualization of what the circle looks like?

S : Yes.

T : Do you think the information provided by Chat GPT was consistent with the given problem?

S : Yes, it was consistent. I pictured the problem, so it immediately provided the answer. I am confident that there were no errors because the result from Chat GPT was the same as what I thought.

It should be noted that Rafayna did not iterate or verify AI answers. This attitude reflects complete dependence on AI and the absence of a critical attitude.

Participant 5's (Adine's) Work Without AI

Based on the analysis of Adine's work in figure 17, it was found that only the analytical ability was met. She did not meet the interpretative ability which concludes one thing that she was not able to find or resolve the answer to the problem.

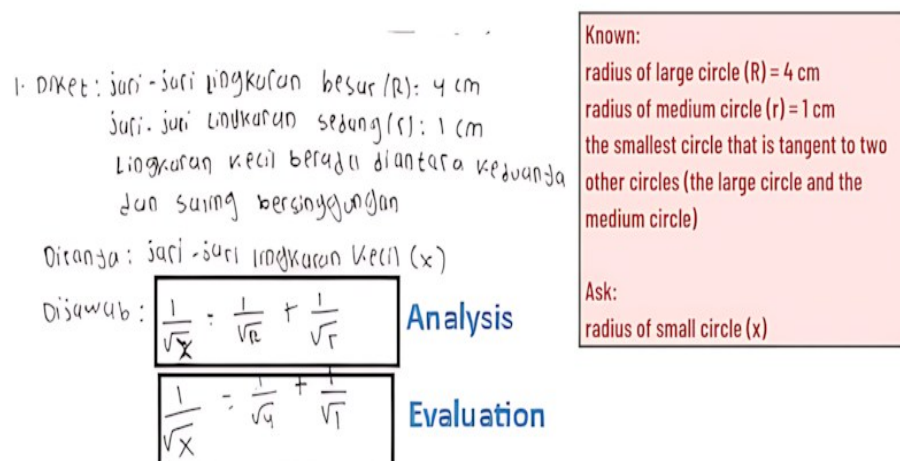


Figure 17. Adine's Work without Using AI

This statement is supported by the following interview results (Note: T = Teacher, S = Student)

- T : What information did you get from the following problem?
 S : What I know is that the radius of the large circle was 4 cm, then the radius of the medium circle was 1 cm, and there was a small circle located between the two circles, tangent to both.
 T : In your opinion, what's the most suitable way to solve this?
 S : The one I wrote down, Sir/Ma'am.
 T : Regarding the method that you used, did you use it randomly, or had you tried it before, so you already knew?
 S : Just trial and error.

From the interview results, it was found that Adine experienced difficulties in the evaluation stage as she was unable to identify an appropriate problem-solving strategy.

Participant 5's (Adine's) Work Using AI

Based on the analysis of Adine's work using AI assistance, it was found that the AI could answer several prompts that she submitted, and it could not explain the reasoning behind each step of the work. The AI also confirmed that the strategy she used was correct. As shown in Figure 18.

Diket:

- Jari-jari lingkaran besar: 4 cm
- Jari-jari lingkaran sedang: 1 cm
- Lingkaran kecil berada diantara lingkaran besar dan sedang dan semuanya saling bersinggungan.

Ditanya?
 Berapakah jari-jari lingkaran kecil.

Ditanyakan:

Analysis

Evaluation

Known:

- radius of large circle = 4 cm
- radius of medium circle = 1 cm
- the smallest circle that is tangent to two other circles (the large circle and the medium circle)

Ask:

- radius of small circle.

Figure 18. Adine's Work with Using AI

- T : Did the AI provide you with a visualization of the problem?
 S : Yes, using words.
 T : Did the AI provide the answer consistent with your method?
 S : Yes, Sir/Ma'am. It provided the full method until the answer was found.
 T : So, when you were looking for it, did you search once or several times?
 S : Just once, and I was sure it was correct because the answer was the same.

Based on the interview results, it was found that the AI confirmed the solution strategy that she used. However, Adine did not perform further iterations after seeing the answer provided by the AI.

In the context of critical thinking, Adine demonstrated fulfillment of interpretation and analysis indicators, but she was weak in evaluation and inference indicators. Adine did not question the validity of the method used by the AI and did not explore other alternative solutions that should have been generated when using AI.

Iteration Process With AI Strengthening Mathematical Idea Construction

Lisa's Iteration Patterns: Performing 3-4 iterations, Resolving Problems Using Different Strategies

Based on the analysis explained in Theme 1, it was found that Lisa performed 3-4 iterations when interacting with the AI due to differences in formula usage in the solution. From the image in Theme 1, it is evident that the iterations show a pattern of progressive refinement:

1. Initial Iteration, in the form of a general exploration regarding methods that can be used to solve the problem
2. Middle Iteration, in the form of critical questions about how the solution/formula provided by the AI originated and can be used to solve the problem.
3. Final Iteration, in the form of confirmation and validation; through which she had reached a conclusion considered valid

This statement is supported by the following interview results. (Note: T = Teacher, S = Student)

T : *So how did the AI answer the question you gave it?*

S : *It gives it step by step. Actually, he wrote the formula skipping a lot of steps. That's why I asked several times where it came from, where it came from. Because he didn't explain. He simplified from Pythagoras, but it became a formula like this.*

Based on the interview, it appears that Lisa perceived the iteration process as a means to gain a deeper understanding of the steps provided by the AI, rather than as mere repetition. From the image in theme 1, each iteration that Lisa performed leads to more specific questions regarding the solution provided by the AI.

Ethak's Iteration Pattern: Performing 7-8 Iterations, Not Resolving Problems

Based on the analysis explained in theme 1, it appears that Ethak performed 7-8 iterations when interacting with the AI. The iteration pattern observed shows a progression from complete ignorance regarding the strategy to be used to solve the problem, towards understanding and validation. From the analysis in theme 1, the following pattern can be spotted:

1. Initial Iteration, involving the reception of two different pieces of information from the AI regarding the method used to solve the problem
2. Middle Iteration, involving the process of identifying the given solution and performing logical validation
3. Final Iteration, involving the validation of a consistent conclusion based on logic

This statement is supported by the following interview results. (Note: T = Teacher, S = Student)

T : *So, what methods did the AI provide to solve the problem? How many were there?*

S : *There are two. Both use the property of tangent circles. But it seems the first property is wrong. The second one also uses the property of tangent circles. But the difference is that it ends up being in the root.*

T : *So, from the method given by the AI, are you sure that method is correct?*

S : *Quite sure because the smallest circle means its radius is smaller than the medium circle and the largest circle. And here it gets 0.44, which is below 1 and 4.*

Based on the interview, it is evident that Ethak perceived the iteration process as a means to understand the steps provided by the AI more deeply, rather than as mere repetition. From the image in theme 1, each iteration performed by Ethak leads to more specific questions related to the solution provided by the AI, and it appears that Ethak's iterations facilitate the construction of personal understanding through a combination of AI and self.

Raka's Iteration Pattern: Performing 0 Iteration, Resulting Incorrect Answers to the Problems

Based on the analysis explained in theme 1, it is known that Raka did not perform any iterations after receiving an answer from the prompt given to the AI. This pattern is problematic because the participant, who previously had an incorrect answer, did not iterate to understand the differences when answered by the AI.

This statement is supported by the following interview results. (Note: T = Teacher, S = Student)

T : *Is the method given by the AI consistent with how you solve the problem?*

S : *Very different*

T : *Do you think there are any errors in the solution given by the AI, such as an incorrect formula or calculation?*

S : *As far as I've used AI, I haven't found any errors in the AI's answers. I am confident with the answer provided by the AI, because I believe this time the answer makes more sense than my previous answer.*

Based on the interview, it appears that Raka did not realize that the iteration process is a way to understand the steps provided by the AI more deeply. Instead, he saw it as a solution-providing machine. From the analysis in theme 1 and the interview results, it is known that Raka only focused on the correct answer. He did not reflect on errors, and; therefore, did not know why the solution he used previously was incorrect.

Rafayna's Iteration Pattern: Performing 0 Iteration, Previously Unable To Answer the Questions To Solve the Problems

Based on the analysis explained in theme 1, it is known that Rafayna did not perform any iterations after receiving an answer from the prompt given to the AI. This pattern is problematic because participants who previously had no solution for the given problem, when provided with a solution by the AI, merely accepted it passively, resulting in no construction of ideas whatsoever.

This statement is supported by the following interview results. (Note: T = Teacher, S = Student)

- T : *In your opinion, is the information provided by ChatGPT consistent with the given question?*
S : *Consistent*
T : *In your opinion, are there any errors in the answer provided by ChatGPT?*
S : *No, not really, because the results from ChatGPT make sense and align with what I thought, so it must be correct.*

Based on the interview, it appears that Rafayna did not realize that the iteration process is a way to understand the steps provided by AI more deeply. She saw it as a solution-providing machine. Rafayna merely became a passive recipient and felt successful because she could solve the problem; in this process, no critical evaluation of the AI's output occurred for the participant.

Adine's Iteration Pattern: Performing 0 Iteration, Giving Answers and Solution Strategy Identical to What Were Provided by AI

Based on the analysis explained in Theme 1, it is known that Adine did not perform any iterations after receiving an answer from the prompt given to the AI. This pattern is problematic because the participant, who previously had an answer to the given problem, did not perform any justification when provided with an answer by the AI using the same formula, and instead used the AI as a tool to correct their answer.

This statement is supported by the following interview results. (Note: T = Teacher, S = Student)

- T : *Is the method provided by the AI consistent with the method you used previously?*
S : *Consistent, using the same formula too*
T : *Did you ask the AI again why it gave that solution?*
S : *No, because its method was the same as mine, so it must be correct*

Based on the interview, it appears that Adine did not perceive the iterative process as a means to gain a deeper understanding of the steps provided by the AI, but merely as a tool to correct answers. Adine was merely a passive recipient and felt successful because her answer and the AI's answer were the same. In this process, there was no exploration of alternative methods or critical evaluation; the participant simply accepted the solution without justification.

Critical Evaluation of AI Output Developing Evaluation Indicators

Lisa's Critical Evaluation: Questioning Validity and Demanding Justification

Based on the analysis of theme 2, Lisa demonstrated an evaluative attitude when interacting with the AI. The evaluation pattern can be observed from the critical questions she posed and is supported by the results of the following interview (Note: T = Teacher, S = Student)

- T : *Did the AI provide information to solve the problem?*
S : *For example, there were two formulas, but I used this one. It said: capital R is equal to the radius of the small circle being sought. R1 is the radius of the large circle, and R2 is the radius of the medium circle. It gave two methods earlier, one using Pythagoras, and the other one, I actually asked where the formula came from, but it didn't answer*
T : *So, how did the AI answer the problem you gave it?*
S : *It gives it step by step. Actually, it wrote the formula with a lot of jumps. That's why I asked the AI several times where it came from, where it came from. Because it didn't explain. It simplified from Pythagoras, but how did it become a formula like this?*
T : *Do you think AI helps you solve problems? Looking at the steps*
S : *It does help, but there are some sudden jumps. That's why I asked about it.*

Apart from that, in figure 19 and figure 20, Lisa can be seen asking AI a follow-up question, in the form of the origin of the formula used by AI and the steps in the solution provided by AI.



Figure 19. Lisa's Prompt Asking the Origin of the Formula Used by the AI

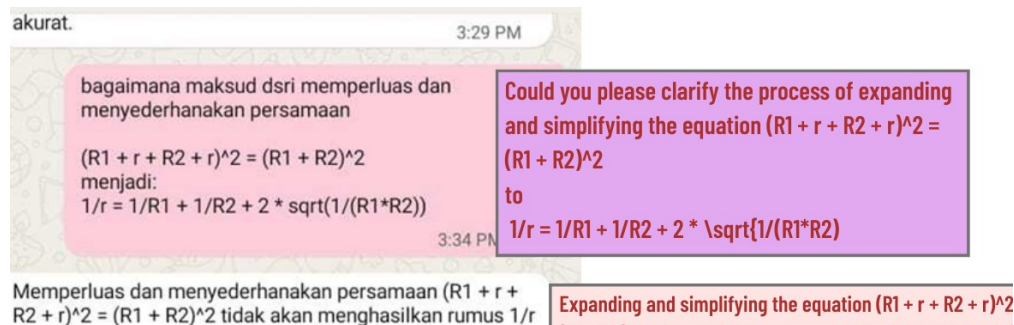


Figure 20. Lisa's Prompt Asking the AI to Clarify the Expansion and Simplification of the Equation

These questions indicate an evaluative process, where Lisa realized that the problem-solving strategy provided by the AI differed from the strategy she used. At this stage, Lisa did not immediately accept the answer given by the AI, but rather evaluated the validity of the solution.

Ethak's Critical Evaluation: Identification of Inconsistencies and Logical Validation

Based on the analysis in theme 2, Ethak demonstrated an evaluative attitude when interacting with AI. The evaluation pattern can be seen from the critical questions asked based on the image and is supported by the following interview results (Note: T = Teacher, S = Student)

- T : So, what methods did the AI provide to solve the problem? How many were there?
 S : There are two. Both use the property of tangent circles. But it seems the first property is wrong. The second also uses the property of tangent circles. But the difference is that the result involves a root.
 T : Do you think there's anything wrong with the method provided by the AI?
 S : Here, the second method wasn't present. Only the first one was wrong because it got 2 or minus 2. A radius of minus 2 is impossible for a circle. And if the result is 2, it's also wrong because it's bigger than the medium circle. Whereas it's supposed to be a small circle.
 T : How did you ensure that the answer provided by the AI was correct?
 S : From the two experiments, the answers were different, and that was very clear from the size of the circles. Since the smallest circle was being sought, it was clear that it would be smaller than the medium circle. So the result must be smaller than 1 cm.

From the interview, Ethak received two different answers from the AI for the same problem. At this stage, Ethak constructed evaluation criteria based on logic, which means that the participant did not act as a passive recipient but instead conducted a self-evaluation to ensure that the answer provided by the AI was logical and valid.

Raka's Critical Evaluation: No Evaluation, No Comparative Analysis

Based on the analysis of theme 2, Raka did not demonstrate an evaluative attitude when interacting with AI. There was no pattern of evaluation regarding the solutions provided by the AI. This statement is supported by the following interview results (Note: T = Teacher, S = Student)

- T : Is the method given by the AI consistent with the method you used for the problem?
 S : It's very inconsistent
 T : Do you think there are any errors in the solution provided by the AI, such as an incorrect formula or calculation?
 S : As far as I've used AI, I haven't found any errors in the AI's answers. I am confident in the answer provided by the AI, because I think this time the answer makes more sense than my previous answer.

From the interview results, it appears that Raka's answer before using AI differed from the answer provided by the AI. At this stage, Raka should have identified the solution and compared the methods of resolution. However, the participant merely replaced the correct answer without understanding the answer provided by the AI, thus, no evaluative construction process occurred.

Rafayna's Critical Evaluation: No Evaluation, Blind Trust

Based on the analysis of theme 2, Rafayna did not demonstrate an evaluative attitude when interacting with AI. There was no pattern of evaluation regarding the solutions provided by the AI. This statement is supported by the following interview results (Note: T = Teacher, S = Student)

- T : How did the AI answer the questions you gave it?
S : I photographed the question directly and immediately got the result
T : In your opinion, does the answer provided by the AI match the problem in the question?
S : Because the AI's result is the same as what I thought.

Before working with AI assistance, the participant did not have their own answer. The interview results show that the participant made no effort to evaluate the given answer, nor were there any critical questions regarding the process or steps for solving the problem. Rafayna accepted the answer without any evaluation.

Adine's Critical Evaluation: Confirmation Without Critical Evaluation

Based on the analysis in theme 2, Adine did not demonstrate an evaluative attitude when interacting with the AI. There was no pattern of evaluation regarding the answers provided by the AI. This statement is supported by the following interview results (Note: T = Teacher, S = Student)

- T : Does the method provided by the AI match the method you used previously?
S : Yes, it matches, it uses the same formula too
T : Did you ask the AI again why it gave that solution?
S : No, because its method is the same as mine, so it must be correct

Before working with AI assistance, the participant had the same answer as the one provided by the AI. The interview results show that the participant made no effort to explore other solution methods. Adine did not compare her own method with the AI's method because it was considered definitively correct; in her usage, Adine only used the AI for confirmation, not evaluation.

Prompt Formulation Developing Explanatory Skills

Participant 1 (Lisa): Evolving Prompts, Performing Iterations

Based on the image analysis, there are several stages in Lisa's prompt formulation, moving from general to specific questions.

In stage 1, the prompt used was not yet specific and was still in the exploration phase which means that stage 1 was a baseline prompt in the form of a question about how to solve the problem.

At stage 2, the prompt was evaluative. At this stage, Lisa identified parts of the AI explanation that were confusing or poorly understood. The prompt in stage 2 was specific, asking about the origin of the formula used. As shown in Figure 21



Figure 21. Lisa's Stage 2 Prompt: Questioning the Origin of the Formula Given by the AI

In stage 3, the prompt used was more of an analysis prompt. Figure 22 shows the prompt that Lisa entered, providing a step-by-step explanation of the formula she needed.

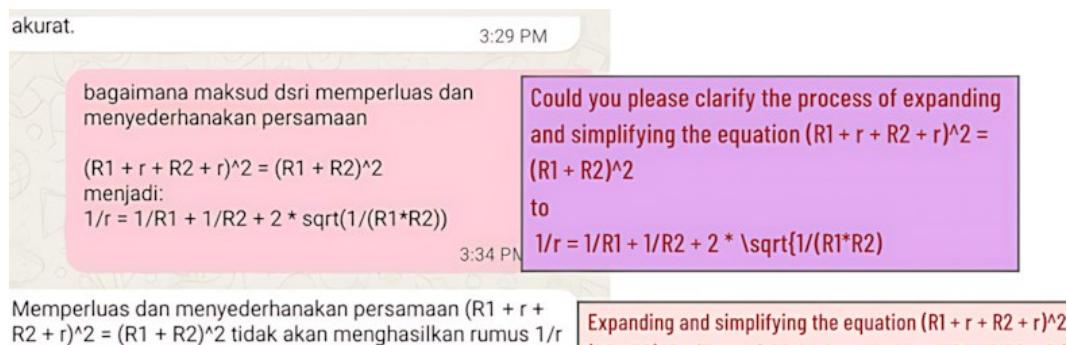


Figure 22. Lisa's Stage 3 Prompt: Requesting a Step-by-Step Explanation of the Equation

At stage 4, the prompt used was more directed towards confirming one's own understanding of the solution provided by the AI. At this stage, Lisa requested validation of her understanding from the AI.

- T : How do you ensure that the answer provided by this AI is correct?
 S : Also, because I saw that all the answers were like mine, too. I definitely saw the triangle, and I think it makes sense. Right, so from the explanation, I just get it.
 T : Does AI help you to understand the meaning of the question and how to solve it?
 S : Yes. Because after asking a few, the explanation given made sense.

Each stage shows an increase in explanation with a prompt evolution pattern starting from general – specific – procedural – confirmation.

Participant 2 (Ethak): From Confusion to Clarity

Based on image analysis, the prompt evolution pattern differs at each stage.

In stage 1, the prompt used was not yet specific and was still in the exploration phase which means that stage 1 was a baseline prompt in the form of a question related to how to solve the problem.

In stage 2, the prompt given was evaluative. This prompt showed the question “why is there a difference between the working steps and the explanation provided by the AI?” as shown in Figure 23.



Figure 23. Ethak's Prompt Questioning the Inconsistency Between the Formula and the AI's Working Steps

In stage 3, the prompt used was more geared towards an analysis prompt. At this stage, Ethak included a prompt in the form of his explanation regarding the method provided and the working steps, as shown in Figures 24 and 25.

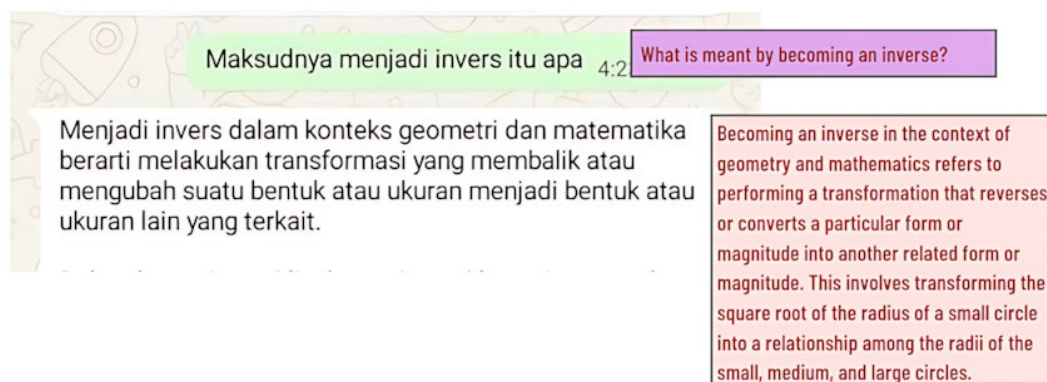


Figure 24. Ethak's Stage 3 Prompt: Asking the Meaning of Inversion in the AI's Solution

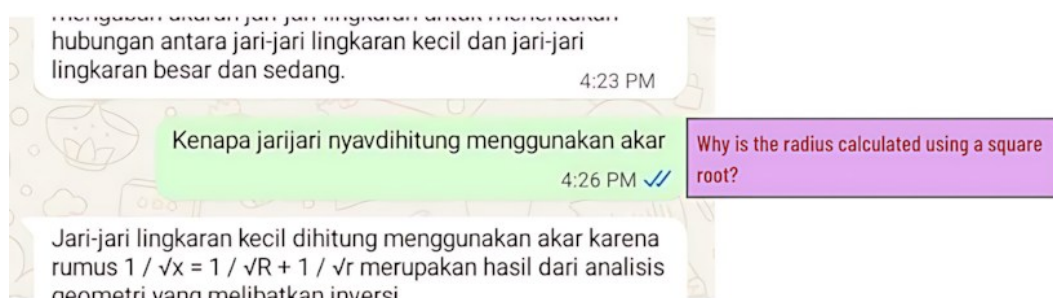


Figure 25. Ethak's Stage 3 Prompt: Asking Why the Radius Is Calculated Using a Square Root

Each stage shows an increase in explanation, with a prompt evolution pattern starting from help seeking, through confusion, to logical reasoning, and reasoning. Validation was carried out by logical thinking based on the results provided by the AI.

Participant 3 (Raka) and Participant 4 (Rafayna): No Further Prompts After Output

Based on the analysis results for theme 1, theme 2, and theme 3, Raka and Rafayna did not provide further questions to the AI regarding the solution provided. They only acted as passive recipients and did not evaluate the solution provided by the AI. This indicates poor explanation development ability

Participant 5 (Adine): Minimal Prompting

Participant 5 only used the initial prompt, which was the given question and was general in nature. After receiving an output identical to her answer, Adine did not ask specific questions regarding the solution, thus there was no development of explanatory skills.

AI Output Verification Strengthening Self-Regulation and Ownership of Answers

Participant 1 (Lisa): High Verification, Repeated Iterations

Based on the analysis from theme 1 to theme 4, it is evident that Lisa verified the AI output by comparing the solution provided by the AI with her own solution. Lisa identified discrepancies by asking about the origin of the formula provided, based on the AI's interpretation.

In the process, Lisa also asked how the formula provided by the AI became so simple, unlike the formula she used. Lisa also performed iterations to check each step of the AI's work. This form of verification helped Lisa understand the explanation provided by the AI explicitly and validly.

Participant 2 (Ethak): Verification as Self-Correction

Based on the analysis from theme 1 to theme 4, it is evident that Ethak verified the AI output by performing several iterations for two solutions with different results provided by the AI. In this case, the participant did not immediately accept and compare both solutions.

In the process, Ethak applied his construct of verification criteria by identifying all solutions provided by the AI, then validating the answers that he deemed consistent with those criteria.

Participant 3 (Raka) and Participant 4 (Rafayna): No Verification

Based on the analysis from theme 1 to theme 4, it was observed that Raka and Rafayna made no effort to validate the information provided. They believed the solutions provided by the AI were entirely correct. Raka and Rafayna did not develop independent verification skills and felt they were correct without understanding the explanations given.

Participant 5 (Adine): Minimal Verification

Based on the analysis from theme 1 to theme 4, it was observed that the process between the participant and the AI was limited to surface-level checking. There was no verification stage for the execution process, alternative solutions, or an in-depth understanding related to the steps of the execution. The process that occurred at this stage was answer confirmation.

From participant 1 to participant 5, it can be seen that AI helped to deepen and strengthen existing idea construction, not just providing answers. In Lisa's case, AI could strengthen interpretation and analysis indicators through external confirmation and validation, which strengthens cognitive confidence. The use of

AI also brings forth indicators of evaluative critical thinking, characterized by numerous iterations performed to evaluate the validity of AI information. Lisa's iteration process identifies the development of inferential abilities, namely considering alternatives, questioning evidence, and drawing conclusions. Lisa used AI as her thinking partner, not as a solution-providing machine.

In Ethak's case, AI helped overcome cognitive barriers that could not be overcome independently. After being given a solution, Ethak performed iterations as a form of critical evaluation; Ethak also performed identification within the AI output. AI encourages mathematical inference performed using relational logic with mathematical reasoning. The construction of ideas carried out between Ethak and AI occurred through dialogue or a two-way conversation. Conversely, in the cases of Raka, Rafayna, and Adine, AI could not facilitate idea construction because the participants did not perform iterations and identification on the solutions provided. Participants simply became passive recipients and were easily trusting of what AI provided, because the solutions provided by AI were definitely correct.

It is concluded that the use of AI in constructing students' mathematical ideas can support students' critical thinking skills. Furthermore, AI-assisted teaching will become more personalized and interactive because AI adapts to the conditions and learning ability levels of students. The integration of AI in mathematics learning will greatly help students understand mathematics lessons better, as AI provides an efficient and flexible learning platform, enabling students to ask questions continuously without hesitation when there are materials or explanations they do not fully understand

DISCUSSION

In this study, the researchers examined how AI can aid students' mathematical idea construction from a critical thinking perspective. Analysis of five students revealed diverse interaction patterns, ranging from passive acceptance to critical evaluation through repeated iterations. These differing patterns represent different levels of critical thinking, requiring students to evaluate the AI's arguments, decide on their further use, determine the information derived from the AI's responses or feedback, and write their final answers (Yoon et al., 2024). Independent evaluation and decision-making reflect students' reluctance to accept AI arguments without critical consideration.

This statement is supported by Vygotsky's theory of social constructivism, where AI acts as a cognitive scaffolding that provides tailored educational experiences to help students achieve tasks they cannot complete independently (Cai et al., 2025). When Lisa and Ethak engaged in repeated iterations, they were actively constructing knowledge. This iteration process aligns with a critical thinking framework that includes the stages of analysis, evaluation, and decision-making (Yoon et al., 2024). Like students with strong metacognitive awareness, they were able to monitor their own understanding, recognize ambiguity in information and use metacognitive strategies such as goal setting and planning, and reflection to obtain desired feedback. This differs from students who engage in minimal iterations, who tend to focus on completing tasks without in-depth understanding. In a collaborative learning context, AI can act as an expert participant that can mediate discussions, provide feedback, and contribute meaningfully to enhancing the learning experience (Cai et al., 2025).

Differences in iteration patterns occur through three main interrelated factors. First, mathematical self-efficacy influences students' courage in questioning AI answers. Students with high self-efficacy have the confidence to identify discrepancies between their understanding and AI explanations, so they will continue to ask questions until they obtain a satisfactory explanation (Güner & Erbay, 2021). Second, students' perspectives on mathematics. Students who view mathematics as a process of building understanding tend to engage in repeated iterations, while students who view mathematics as a collection of procedures tend to be immediately satisfied with AI's solution steps (Schukajlow et al., 2023). Third, AI literacy, formed from learning experiences with digital technology, determines students' ability to interact critically with AI (Güner & Erbay, 2021; Schukajlow et al., 2023). These three factors indicate that the effectiveness of AI use in learning depends not only on technology but also on students' thinking and feelings.

The results of this study align with those of Lee and Yeo (2022), who demonstrate that AI-based collaborative learning systems improve students' problem-solving abilities and foster social learning by providing real-time feedback. However, this study extends this understanding by demonstrating that feedback needs to be followed by responses through an iterative process. The fundamental difference

lies in the emphasis on student agency. Unlike previous studies that focused on the effectiveness of AI systems in generating correct solutions, this study explores how students actively construct their understanding through repeated dialogue with AI. This study also complements the findings of Chau et al. (2025) regarding the creation of prompts that strengthen student engagement in the analysis and evaluation stages. Unlike Holmes et al. (2022), who position AI as an adaptive tutoring system, the researchers position AI as a conversational partner that facilitate reflection and the construction of mathematical knowledge. This finding is important because it suggests that learning design with AI should not only focus on the accuracy of answers but also on the quality of cognitive processes facilitated by the interaction between students and AI (Kasneci et al., 2023).

Theoretically, the findings suggest that AI functions as a cognitive scaffolding that supports students operating within Vygotsky's Zone of Proximal Development (ZPD). The integration of AI in mathematical learning has great potential to improve pedagogy and manage cognitive load by simplifying complex mathematical concepts through structured explanations, thus making the learning process more efficient (Patac & Patac, 2025). Furthermore, AI creates a collaborative learning environment between students and machines, where students can work together with AI to solve given problems through an iterative process that facilitates the development of thinking skills by providing information related to a specific topic, highlighting unexplored aspects, and introducing new areas within the topic (Almarashdi et al., 2024). This study adds to the literature on metacognition in digital learning by demonstrating that iteration with AI serves as an external metacognitive prompt that encourages students to continuously monitor and evaluate their understanding.

Pedagogically, this research offers significant practical implications for mathematics educators. AI integration has been shown to catalyze effective interactive and feedback-based learning to enhance critical thinking (Panit, 2024). AI feedback gives students a direct opportunity to reflect on mistakes and improve their thinking processes, reinforcing the iterative nature of learning (Panit, 2024). Teachers need to design tasks that explicitly ask students to explain their reasons for accepting or rejecting AI solutions with the goal of encouraging students to iterate with AI. This iteration pattern can be used as an indicator of formative assessment, where students with minimal iteration may need additional support so as not to immediately accept the information provided by the AI without further evaluation. In addition, AI as a self-learning platform can reduce emotional barriers because students feel freer to ask questions repeatedly without fear of being judged (Gouia-Zarrad & Gunn, 2024), with the advantages of instant solutions, fast feedback, and ease of access without requiring special technical skills (Wardat et al., 2023). However, because the quality of AI feedback depends on the quality of the prompts provided by students (Egara & Mosimege, 2024), learning needs to include explicit instruction on AI literacy, including effective prompt engineering strategies, verification of AI output, and metacognitive monitoring during interactions.

CONCLUSION

Based on the analysis of five participants with different cognitive characteristics, it can be concluded that AI facilitates the construction of mathematical ideas in terms of critical thinking indicators, through AI facilitation mechanisms, namely cognitive scaffolding. This study identified a spectrum of different interaction patterns, ranging from passive acceptance with minimal iteration to critical evaluation through repeated iterations. These differences in patterns are influenced by three main interrelated factors, namely mathematical self-efficacy, students' perspectives on mathematics lessons, and AI literacy developed through previous learning experiences. AI is confirmed to provide correct interpretations and strategies and multiple solutions that trigger critical evaluation, as in the case of Ethak, who received two different solutions resulting in a comparative analysis process. Cognitive development must be built through dialogue between students and AI through Iteration and Refinement, where AI provides space for students to reflect on their mistakes, so they can immediately improve their thinking processes and create cognitive exploration. Theoretically, this study enriches the metacognitive literature in digital learning by showing that iteration can encourage students to continuously monitor and evaluate their understanding. It is important to note that the iteration that occurs is not just a mechanical repetition but must be accompanied by an active attitude in questioning, verifying, and validating the output provided by AI. The right prompt will guide the AI to

provide appropriate feedback, the number of prompts/iterations written will help students get valid understanding, not just the correct answer.

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Information

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