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Penggunaan Pendekatan Konkret-Gambar-Abstrak untuk Meningkatkan Pemahaman Konseptual Peserta Didik tentang Pecahan

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Abstrak

Dalam makalah ini, kami mengeksplorasi bagaimana penggunaan pendekatan Konkret-Gambar-Abstrak (CPA) untuk meningkatkan pemahaman konseptual siswa tentang pecahan di Distrik Capricorn Utara, Afrika Selatan. Penelitian ini berlandaskan pada teori pemahaman konseptual yang berakar pada konstruktivisme kognitif. Pemahaman konseptual didefinisikan sebagai pemahaman tentang ide-ide matematika sebagai sistem yang terintegrasi dan fungsional dengan tiga komponen: pemahaman, operasi, dan hubungan. Desain penelitian metode campuran tertanam satu fase yang dilakukan secara bersamaan dengan mengumpulkan data kualitatif melalui wawancara berbasis tugas dan data kuantitatif melalui *pretest* dan *post-test*. Analisis tematik deduktif digunakan untuk menganalisis data kualitatif, sedangkan data kuantitatif dianalisis menggunakan statistik deskriptif serta uji T sampel independen dan berpasangan. Sebanyak 70 siswa Kelas 7 yang berpartisipasi sebagai sampel dalam penelitian ini. Temuan menunjukkan bahwa pendekatan CPA meningkatkan pemahaman konseptual siswa tentang pecahan, meskipun ukuran efek intervensi minimal dan dapat diabaikan. Selain itu, penelitian ini menemukan bahwa penggunaan objek konkret dan alat bantu visual terbukti efisien. Hasil kualitatif juga mengungkapkan bagaimana pemahaman konseptual siswa tentang pecahan meningkat selama intervensi. Studi ini berkontribusi pada metode pengajaran inovatif tentang pecahan, yang berpotensi meningkatkan kinerja matematika dan menginspirasi para pendidik untuk mengeksplorasi pendekatan pengajaran baru.

Kata Kunci: desain tertanam bersamaan satu fase, pecahan, pemahaman konseptual, pendekatan konkret-gambar-abstrak, pendekatan metode campuran

The Use of Concrete-Pictorial-Abstract Approach to Enhance Learners' Conceptual Understanding of Fractions

Abstract

In this paper, we explored how the use of the Concrete-Pictorial-Abstract (CPA) approach enhances learners' conceptual understanding of fractions in the Capricorn North District of South Africa. The research was framed within the conceptual understanding theory rooted in cognitive constructivism. Conceptual understanding refers to understanding mathematical ideas as an integrated and functional system with three components: comprehension, operations, and relations. A one-phase concurrent embedded mixed-method research design was used to gather qualitative data through task-based interviews and quantitative data through pretests and post-tests. Deductive thematic analysis was used to analyse the qualitative data. The quantitative data was analysed using descriptive statistics, independent and paired samples T-tests. Seventy (70) conveniently sampled Grade 7 learners participated in the study. The findings indicate that the CPA approach enhanced the learners' conceptual understanding of fractions though the effect size of the intervention is minimal and negligible. Furthermore, the study found that the incorporation of concrete objects and visual aids proved to be efficient. Additionally, the qualitative results revealed how learners' conceptual understanding of fractions was enhanced during the intervention. This study contributes to innovative teaching methods of fractions, potentially enhancing mathematics performance and inspiring educators to explore new teaching approaches.

***Keywords**: concrete-pictorial-abstract approach; conceptual understanding; fractions; mixed method approach; one-phase concurrent embedded design*

INTRODUCTION

Fractions are an integral component of elementary and secondary mathematics. They incorporate percentages, decimal numbers, and proportions. According to Department of Basic Education (2018), learners in the senior phase (Grades 7-9) should develop a conceptual understanding of fractions. Fractions are the fundamental building blocks for a strong conceptual grasp of algebra and other mathematical ideas. Learners with a conceptual understanding of fractions will have a solid foundation for all other topics such as trigonometry, calculus, geometry, and abstract mathematics (Makhubele, 2021). According to Bailey et al. (2015), a conceptual understanding of fractions involves understanding their properties, such as the magnitude, relevant principles, and notations for expressing them. Kilpatrick et al. (2001) showed that learners' conceptual understanding enables them to comprehend, operate and establish relations among different mathematics concepts.

According to Kilpatrick et al. (2001), comprehension of Mathematics refers to the ability to understand and interpret mathematical concepts, problems, and solutions. Therefore, comprehending fractions enables learners to define them from their five key constructs: part-whole, measure, operator, quotient, and ratio (Zhang, 2016; Getenet & Callingham, 2017). The part-whole construct entails splitting an object into two or more equal parts, for example, the fraction $\frac{3}{5}$. This means three parts were taken from a whole of five equal parts. The ratio construct entails the comparison of two multiplicative and relative quantities. For example, the ratio of three cats to the total of eight pets at the pet store could be represented as 3:8. The operator builds on the concept that a fraction can be a multiple of a unit fraction, for example, $3/4 \ \ of\ 30\ kilometres = 22{,}5\ km$. The quotient construct refers to fractions as a result of division (a/b is equivalent to a÷b). The measure construct of fractions identifies a unit and then uses that unit to determine the units of other objects (Van de Walle et al., 2020). For example, the unit of the fraction $\frac{9}{10} = \frac{1}{10}$ therefore $\frac{9}{10} = \ 9\ times\ \frac{1}{10}\ units$. Conceptual understanding involves competence with operations involving fractions such as addition, subtraction, multiplication, and division. The correct use of operations in a fraction activity is evident when learners can develop mathematical discourse, which plays a vital role in the development of conceptual understanding (Cobb et al., 2003). For example, when learners are competent in working with operations, they will be able to establish that $\frac{1}{8}\ is\ half\ of\ \frac{1}{4}\ and\ \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$. Connections refer to various ways in which different mathematical ideas or objects can be related to or linked to each other. According to Maoto et al. (2016), connections are developed from big ideas that are central to mathematical understandings. Connections in fractions refer to the ability to establish relationships within and between fraction constructs such as part-whole, quotient, ratio, measure and operator and representations and the inter-relationship between the constructs (Mills, 2016). Three major fraction models are set area and length models. The area model is based on how a portion of a region or area relates to the entire region or area. The whole is represented as the area of a specific shape. It provides a visual representation of how the whole is divided into parts based on the shape's area (e.g. circular food like pizza). The length model represents fractions on a number line or a linear scale. It helps visualise fractions in terms of their position along a line segment and shows the relationship between fractions and whole numbers (eg music, distance measures using number lines or paper strips). The set model presents the number of discrete items in the whole set and the number of items in the part are used to calculate fractions (e.g. learners in a class or items in a jar.

Teaching and learning fraction concepts continue to be a challenge to both elementary and middle school mathematics teachers and learners (Hunt et al, 2022). For example, in South Africa, Baidoo (2019) and Makhubele (2021) found a persistent trend of poor conceptual understanding of fractions contributing to misconceptions that learners carry to higher grades. This is confirmed by the South African Mathematics Diagnostic Reports (2018–2023) which revealed that matric learners performed poorly in problems involving fractions (DBE, 2018–2023). The Department of Basic Education (2023) revealed that learners struggled to handle the fraction $\frac{1}{x} + \frac{1}{y} = 1$ when solving a simultaneous equation. According to Naidoo and Hajaree (2021), the problem emanates from teachers' conventional approaches when teaching fractions. Conventional teaching approaches are teacher-centred, and the learners participate through listening and writing notes (Teoh et al. 2019). These conventional approaches do not provide learners with opportunities to develop a conceptual understanding of fractions (Lestiana et al., 2016). To help learners develop conceptual understanding, mathematics instruction must be designed to encourage them to build their knowledge through the use of contemporary teaching approaches which engage learners in hands-on activities.

Wiest and Amankonah (2019) firmly recommended that concrete and pictorial representations could enhance learners' conceptual understanding of fractions. According to Sumartini and Priatna (2018), thinking capabilities tend to progress from concrete to abstract thinking patterns. One approach that allows for the development of learners' conceptual understanding and solution of Mathematics problems is the Concrete Pictorial Abstract (CPA) approach (Bruner, 1966). The CPA approach gives meaning to abstract mathematics concepts by presenting them in a physical form which aids the construction of new knowledge (Oseńa & Tesorero, 2024). It further proposes that learners acquire mathematics knowledge first by working with concrete objects, then by using visuals, and only after those to the realm of mathematics dominated by abstract symbols.

In the concrete phase, learners are introduced to the abstract concept with physical, interactive concrete materials. The next stage is the pictorial or visual stage, where problems are represented using visual depictions of concrete items. The pictorial phase prompts learners to link the concrete objects they just interacted with to the theoretical images. Building or sketching a model for a mathematical concept makes it simpler for learners to grasp difficult abstract concepts. At the abstract or symbolic stage, learners use abstract symbols to model mathematical situations or problems. Learners only progress to this stage after they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the use of abstract concepts or symbols. At this stage, only numbers, notation, and mathematical symbols and operations constitute the language of mathematics. Table 1 presents the CPA stages applied in this study.

Table 1. Stages of CPA

Stage	Concrete Stage	Pictorial Stage	Abstract Stage
Explanation	Physical materials representing the mathematical concept being taught were used.	Visual aids such as diagrams, pictures, or graphs were used to represent the concept being taught.	Learners apply mathematical operations and equations.
Example	Learners used materials such as blocks, dough and paper strips.	Drawing pictures of four pals sharing the pizza	Learners move on to using abstract operations like $+ and \times$ to represent these concepts in the test.

This CPA approach has been shown to enhance self-efficacy in mathematics among primary school learners, regardless of gender, by fostering a more inclusive learning environment (Yuliyanto et al., 2019; Akinoso, 2012). In their study, Zhang et al. (2022) found that CPA enhanced Grade 4 learners' proficiency and conceptual understanding of fractions. The CPA approach has shown to be effective in enhancing mathematical learning, particularly in Singapore, where it has been widely implemented (Lutfi & Dasari, 2024). Teaching approaches that use concrete manipulatives are often employed to enhance and support conceptual understanding (Mntunjani et al., 2018; Minarti & Wahyudin, 2019). Hunt et al (2022) used Dream 2B, a web-based universally designed fraction game, which positively impacts learners' conceptual understanding of fractions. As a result, learners with a sound conceptual understanding of common fractions built from real-life scenarios and the use of manipulatives develop meaningful Mathematics and can transfer knowledge to different real-life contexts (Mills, 2016). Despite these benefits, some studies highlight that the CPA approach may not significantly alter learners' overall attitudes or anxiety levels, indicating that its impact can be context-dependent (Salingay & Tan, 2018; Putri et al., 2020). Moreover, pictures may cause extraneous cognitive load (Ollesch et al.,2017). Therefore, this paper sought to explore the use of CPA to enhance learners' conceptual understanding of fractions. The paper sought to respond to the following research questions: How does the use of the CPA approach enhance Grade 7 learners' conceptual understanding of fractions? And What changes in conceptual understanding of fractions do Grade 7 learners have after being exposed to the CPA approach? Furthermore, the following research hypotheses were tested at a 5% significant level with H_1: There is a significant difference between learners' pre-test and post-test scores of fractions and H_2: There is a significant difference between boys' and girls' post-test performance.

METHOD

A one-phase concurrent embedded mixed-method design within a pragmatism paradigm was followed. In this paper, the qualitative data was nested within the qualitative data (Creswell, 2008). Figure 1 shows a diagrammatic representation of the research design.

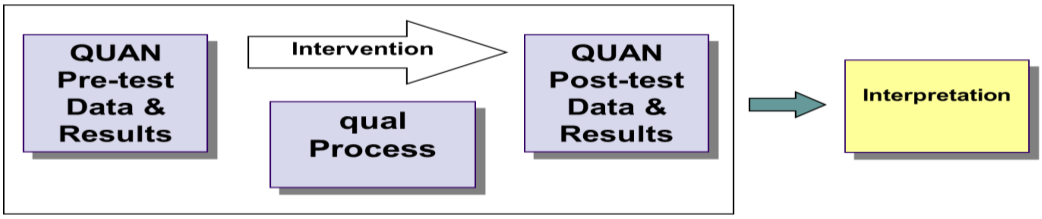

Figure 1. Diagrammatic Representation of Concurrent Mixed Method Design

Convenience sampling was used to select the 75 Grade 7 learners (47 boys and 28 girls) in the Capricorn North District, Limpopo Province, South Africa. The participants were conveniently selected because they were both accessible and willing to participate. Convenience sampling is also purposeful (Nyimbili & Nyimbili, 2024). Therefore, learners were purposefully chosen as well because they are learning fractions which are prescribed for their syllabus. Their ages ranged from 10 to 12 years.

Quantitative data was collected using a pre-test and a post-test and analysed using descriptive statistics (the mean and standard deviation) and inferential statistics (paired sample t-test). The t-test was used to establish whether there were statistically significant differences between a pre-test and a post-test for the same group (Cohen et al., 2018). Cohen's - *d* was used to establish the magnitude of the gain in conceptual understanding because of the intervention. During the qualitative phase, learners were provided with concrete objects to solve the fraction problems. For example, learners were given the following fraction problem to solve: *Four pals are enjoying 10 muffins, ensuring that everyone gets a fair share. How many muffins will each buddy receive?* Each group was given balls of dough and a plastic knife. They were required to find a way to share the muffins fairly within a group of learners (see Figure 3). The qualitative data were collected during the intervention when learners were engaged in these three stages of CPA: concrete, pictorial, and abstract. Learners were observed when translating between concrete objects to pictures and then abstract phase to solve the fractions problems.

Written classwork tasks and task-based interviews were used to collect the qualitative data during the intervention. Classwork tasks generated data as documents, which allowed the researcher to study learners' responses during the CPA approach intervention. While learning fractions, task-based interviews were used to learn about an individual's comprehension, operations, and connection abilities. All interviews were voice-recorded. These data were analysed using deductive thematic analysis where patterns and themes were derived from the theory (Braun & Clarke, 2012).

In phase one, learners were given a pretest out of 25 to assess their understanding before the CPA approach intervention. In the second phase, the CPA approach was implemented, and learners were given a posttest thereafter. The Limpopo Province Department of Education gave their permission. Learners' parents gave consent for their children and learners assented themselves to participate in the study.

RESULTS

Quantitative results

The statistical software package used to analyse quantitative data was Stata Release. Together with summary measures, the results from both independent and paired-sample t-tests are presented. The independent samples t-test was used to compare male and female learners in the pre-and post-tests, while the paired samples t-test was used to compare learners' performances between the pre-and post-tests. The results were interpreted at the 95% confidence interval.

Table 2. Comparisons between Pre-Test and Post-Test Performances
Paired t-test

Variable	Obs	Mean	Std. err.	Std. dev.	[95% conf.	interval]
postt~25	75	14.37333	.5996315	5.192961	13.17854	15.56813
preto~25	75	9	.5663221	4.904493	7.871579	10.12842
diff	75	5.373333	.5656132	4.898354	4.246324	6.500342

```
     mean(diff) = mean(posttotal25 - pretotal25)                     t =   9.5000
 H0: mean(diff) = 0                            Degrees of freedom =        74
 Ha: mean(diff) < 0           Ha: mean(diff)!= 0          Ha: mean(diff) > 0
 Pr(T < t) = 1.0000         Pr(|T| > |t|) = 0.0000         Pr(T > t) = 0.0000
```

The learners' overall performances show a significant enhancement in the post-test ($p < 0.0001$). Specifically, the average post-test performance of approximately 14, 3% is significantly higher than the pre-test average of 9,0 %. The results of the paired samples t-test show that there was an increase of 5.37 in the post-test mean score. Furthermore, the p-value between the pretest and post-test mean is $p=0.000<0.05$, the p-value suggests that there is a statistically significant difference between the pretest and post-test mean scores. Therefore, the null hypothesis that there is no significant difference between learners' pre-test and post-test scores of fractions is rejected. We, therefore, conclude that the use of CPA enhances learners' conceptual understanding of fractions.

Table 3. Effect Size based on Mean Comparison
Number of obs = 150

Effect size	Estimate	[95% conf.	interval]
Cohen's d	-1.063861	-1.404481	-.7200768
Hedges's g	-1.058459	-1.39735	-.7164206
Glass's Delta 1	-1.095594	-1.458079	-.727362
Glass's Delta 2	-1.034734	-1.392694	-.6712139
Point-biserial r	-.4720831	-.5772755	-.3407685

The Cohen's d shows an estimate of -1.063861 indicating a minimal effect size. This suggests that, regardless of whether the difference in mean scores between the two testing conditions is statistically significant, the actual difference between the pre-test and post-test averages is negligible. Therefore, this shows that though CPA enhanced learners' conceptual understanding of fractions, its effect is very small.

Table 4. Gender Comparisons (Pre-test)
Two-Sample T-Test with Equal Variances

Group	Obs	Mean	Std. err.	Std. dev.	[95% conf.	interval]
Female	28	10.5	.9670497	5.117146	8.515778	12.48422
Male	47	8.106383	.6706715	4.597892	6.756391	9.456375
Combined	75	9	.5663221	4.904493	7.871579	10.12842
diff		2.393617	1.145057		.1115216	4.675712

```
    diff = mean(F) - mean(M)                                         t =   2.0904
H0: diff = 0                                   Degrees of freedom =        73

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.9800         Pr(|T| > |t|) = 0.0401         Pr(T > t) = 0.0200
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Table 4 shows the comparison between the boys' and girls' performance in the pre-test. The results indicate no significant difference between girls and boys (t = 2.0904). p = 0.0401). We, therefore, fail to reject the null hypothesis and conclude that both boys and girls performed equally in the pre-test.

Table 5. Gender Comparison (Post-Test)
Two-Sample T-Test with Equal Variances

Group	Obs	Mean	Std. err.	Std. dev.	[95% conf. interval]	
Female	28	15.46429	.9448862	4.999868	13.52554	17.40303
Male	47	13.7234	.765649	5.249025	12.18223	15.26458
Combined	75	14.37333	.5996315	5.192961	13.17854	15.56813
diff		1.740881	1.231422		-.71334	4.195103

```
    diff = mean(F) - mean(M)                                      t =   1.4137
H0: diff = 0                                     Degrees of freedom =       73
    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.9192         Pr(|T| > |t|) = 0.1617          Pr(T > t) = 0.0808
```

Table 5 indicates that the mean difference of 1.740881 between boys and girls learners' performances in the post-test was not statistically significant (p = 0.1617). At t=1,4137, we fail to reject the null hypothesis that boys and girls perform equally in the post-test. This shows that the performances of boys and girls learners were nearly identical in both the pre-test and post-test. This indicates that the intervention does not favour either boys or girls.

Qualitative results

The qualitative data were analysed using deductive thematic analysis. Learners were presented with questions that sought to explore their comprehension, connections and operations as guided by the theoretical framework. The results of the analysis were fitted into the themes guided by the tenets of the theoretical framework: comprehension, connections and operations.

Comprehension

Learners were presented with the following question which sought for their comprehension:

Four pals are enjoying 10 muffins, ensuring that everyone gets a fair share. How many muffins will each buddy receive?

Learners were given a ball of dough and a plastic knife. They were required to find a way to share the muffins among the four learners. In Figure 2, group 2 learners were able to use concrete material in the form of muffins to demonstrate their comprehension of the division of fractions. The learners moved from the concrete to the pictorial stage in Figure 3. They represented the division of muffins amongst the four friends in the form of pictures. Figure 3 shows how each friend is connected to two full muffins and a half.

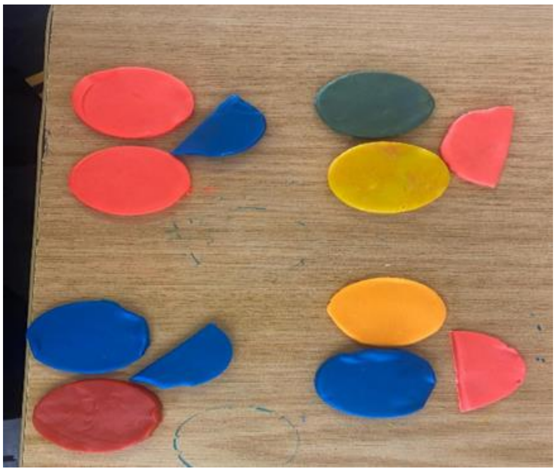

Figure 2. Group 2 Learners Use Dough (Concrete) to Show How 10 Muffins Can Be Shared Amongst 4 Friends

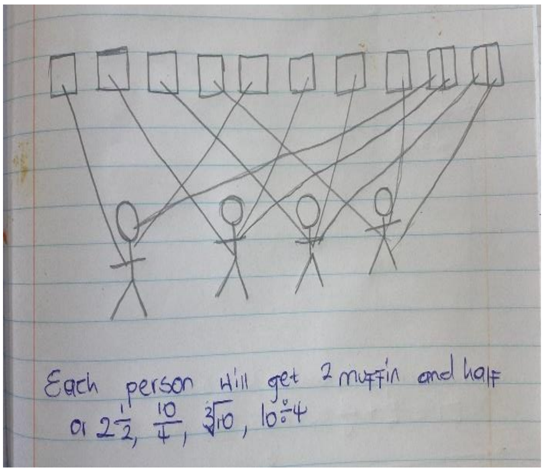

Figure 3. Group 2 Learners' Picture Showing How 10 Muffins Are Shared Between 4 Friends

Most groups of learners were able to translate from concrete material (dough) to a drawing to get the answer. For example, in Figure 2 learners used concrete materials to demonstrate the division of the muffins. This indicates learners' comprehension of the division of fractions because they were able to represent fractions as a part-whole. In Figure 3, learners were able to show fractions as an interrelated construct by representing them as a quotient and a ratio of $\frac{10}{4}$. Furthermore, learners were able to realise that the total number of muffins as an operator of $2\frac{1}{2}$ of 4 as well as a measure of 10 multiplied by $\frac{1}{4}$. To corroborate these findings, the researcher engaged group 2 learners in task-based interviews in this way.

Researcher	:	*Can one of the group members tell us how you managed to solve the problem?*
Learner 3	:	*Initially I had difficulty working the question until me and my friends started to make the muffins and shared them amongst ourselves. We then made a sketch.*
Researcher	:	*How did you use this sketch to solve the problem?*
Learner 2	:	*Sketching the diagram helped us to see how we can share the pizza by cutting it into pieces of the same sizes and sharing them.*
Researcher	:	*How did this assist you in understanding the concept?*
Learner 3	:	*I now understand it even better because after we made the muffins using the dough, we shared them. I could finally see that 2 and a half is the answer. This means that 10 divided by 4 equals two and a half.*

From the interviews, it is observed that the CPA helped the learners to comprehend the division of fractions. Learners moved from the concrete to the pictorial and then to the abstract stage to comprehend the problem. The CPA assisted the learners in applying part-whole, quotient, ratio and measure principles of comprehension of fractions.

Operations

One of the tasks required learners to demonstrate knowledge of operations involving fractions was the use of models such as building blocks to enhance their understanding. The tasks that learners were asked to work on were:

Moloko and Orefile ordered two medium pizzas, one Chicken and one Mushroom. Moloko ate $\frac{5}{6}$ of a pizza, and Orefile ate $\frac{1}{2}$ of a pizza. How much pizza did they eat altogether?

This task required learners to use the addition operation to get the answer. The task needed the learners to add the fractions of pizza eaten. Learners started by using the building blocks as shown in

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Figure 4 to represent the fractions. The building blocks helped them to draw a pictorial representation of the addition in Figure 5.

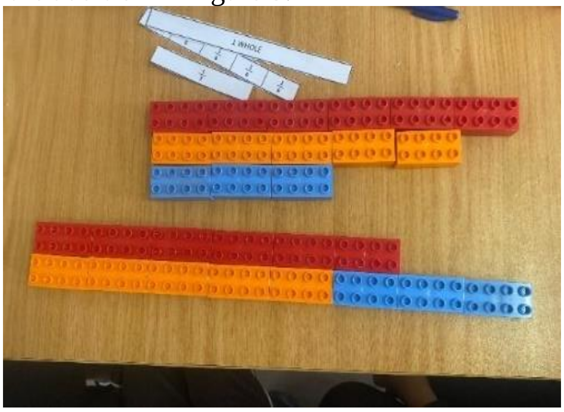

Figure 4. Group 5 Learners' Use of Building Blocks for the Addition of Fractions

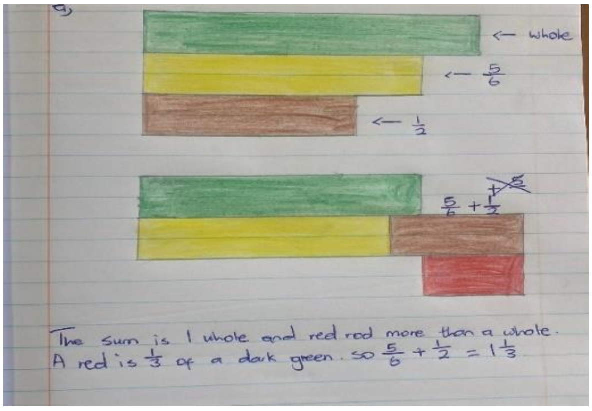

Figure 5. Group 5 Learners' Diagrammatic Representation of the Pizza Model

Figures 4 and 5 show how group 5 learners used the pizza model in the form of building blocks to add fractions and the diagrammatic representation of the pizza model respectively. The learners constructed the model by using the red, orange and blue blocks. The red blocks in Figure 4 and the green colour in Figure 5 represented the whole which is 6 units whereas the orange blocks and the yellow colour represented the 5 of the 6 blocks and the blue blocks and the brown colour represented half of the blocks. The learners applied the measure construct where 1 block represented a unit of the whole (1/6). The addition of fractions was demonstrated by 5 orange unit blocks and 3 blue unit blocks added together and compared to the whole 6. The learners realised that the added blocks (8) were 2 more than the whole blocks (6). The model and the picture allowed the learners to read off the answer as one whole and 2 more blocks. When moving to the abstract level they were enabled to apply addition operation and find the answer as 1 and a third. The researcher interviewed learners and asked them to explain if the models helped them find the answers.

Researcher : *How did the use of models help you to understand operations involving fractions?*

Learner 3 : *The use of blocks and pizza models helps because now I could see and touch the six blocks that represented a whole, 5 blocks out of the whole 6 blocks and half of the 6 blocks. So, it was easy to see what I was adding, 5 and 3 which gave me 8 blocks of* $\frac{1}{6}$ *each.* I also realised that when comparing 8 blocks to whole 6 blocks equal to one whole and 2 blocks. Therefore, what they all ate together was 1 whole pizza and 2 blocks.

From the interviews, learners confirmed that the concrete materials helped them to read off the answer to the given question. This shows that the use of concrete objects made the addition of fractions meaningful and made it easier to work with the operation.

Connections

To demonstrate connections involving fractions, learners were provided with paper strips representing the whole and the fractions for each runner. Learners were given the following fractions for each runner: Makoena: $\frac{3}{4}$ Sello: $\frac{1}{2}$ Maphuti: $\frac{5}{6}$ Shantel: $\frac{5}{8}$ Khensani: $\frac{5}{9}$ Phenyo: $\frac{1}{3}$. The fractions indicate the distance covered by each runner compared to the whole. Learners were asked to respond to the following question:

Can you place these friends on a line to show where they are between the start and finish and determine who is winning?

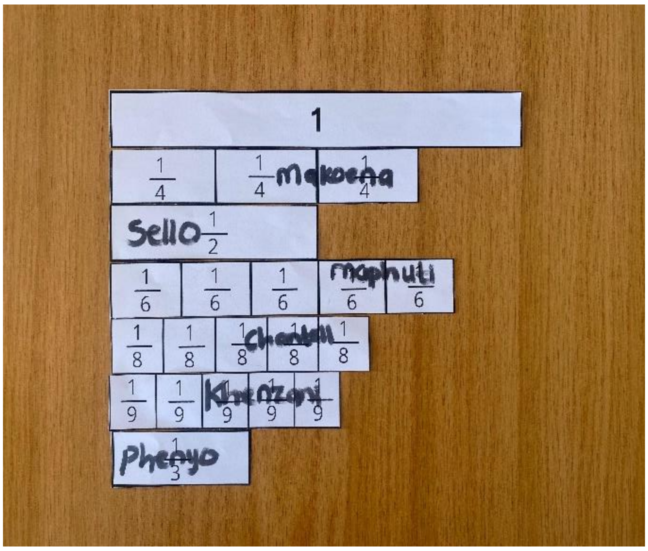

Figure 6. Group 4 Learners Use Paper Strips to Make Connections of Who Is Winning

Figure 6 shows how group 4 learners compared each runner's paper strip against the whole to determine who was winning. The learners developed connections between the fractions and the whole. They were able to apply the length model as well as a measure construct by connecting different kinds of fractions. This is indicated by the placement of the paper strips. Their placement of the strips enabled them to realise that $2 \times \frac{1}{4}$, $3 \times \frac{1}{6}$, $4 \times \frac{1}{8}$ all were at the same length which equals to $\frac{1}{2}$. The learners were also able to notice that $\frac{1}{9}$ is smaller than $\frac{1}{6}$ because three-ninths make $\frac{1}{3}$ whereas two-sixths make $\frac{1}{3}$. Furthermore, they realised that $\frac{1}{8}$ is smaller than ¼ hence two-eighths make $\frac{1}{4}$. They further realised that 3 x $\frac{1}{9}$ equals $\frac{1}{3}$ which is also equal to 2 x 1/6. The learners were also able to make connections between the areas of different fractions. They realised that $\frac{1}{2}$ is greater than $\frac{1}{3}$ which is greater than $\frac{1}{4}$ which is greater than $\frac{1}{6}$ which is greater than $\frac{1}{8}$ and subsequently $\frac{1}{9}$. During the interviews, the learners responded this way.

Researcher	:	*Based on how you have placed the paper strips for each runner, who is winning?*
Learner 2	:	*Mmaphuti is winning.*
Researcher	:	*Why do you say Mmaphuti is winning?*
Learner 2	:	*I said Mmaphuti is winning because I can see on the paper strips that $\frac{5}{6}$ covers more area than all the other fractions. Hence, I am saying Mmaphuti is winning.*

The interview responses corroborated findings from the observations. These revealed that the CPA helped learners to form connections between fractions themselves as well as connections with real-life phenomena. The learners were able to relate the magnitude of fractions with area, length and set models.

DISCUSSION

The study investigated the use of the CPA approach on Grade 7 learners' conceptual understanding of common fractions. The paired t-test results indicate that the use of the CPA approach increased learners' conceptual understanding mean score. The results are consistent with the findings of Nainggolan (2022), who found that the CPA approach led to an increase in conceptual understanding scores from 73.3 to 80.2 among second-grade learners. Similarly, the results concur with Minarti and Wahyudin (2019) who found that learners using the CPA approach, showed a better conceptual

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understanding of mathematics concepts compared with those receiving direct instruction. On the other hand, these findings are inconsistent with Lutfi and Dasari (2024) who found that the CPA approach may not adequately address abstract mathematical concepts, which could hinder deeper conceptual understanding in more advanced contexts. Additionally, the mean score attained by the learners during the pre-test, when analysed about the post-test scores, reveals that it is approximately -0.0367 standard deviations greater than the mean score recorded during the post-test, which subsequently implies a remarkably minimal effect size of a negligible magnitude. These results are inconsistent with Morano, et al. (2020) who found a larger effect size of CPA when used during solution of fractions. The results concur with Salingay and Tan (2018) who found a minimal effect size when using CPA intervention. The findings also indicated that male and female learners exhibited comparable levels of performance. This implies that the intervention does not differentiate between male and female learners. This is in line with Akinoso's (2012) findings that CPA does not segregate between male and female learners. Furthermore, this study concurs with Yuliyanto et al. (2019) who found that both boys and girls benefitted equally from the CPA approach concerning an understanding of mathematics.

During the implementation of the CPA approach, learners acquired comprehension, connections, and the ability to operate on fractions. In the qualitative phase, the use of concrete objects and pictorial representation helped learners to develop comprehension of fractions. Learners were able to apply part-whole, ratio, quotient and measure constructs to fractions. This indicates that the CPA developed the learners' ability to represent fractions as an interconnected concept and interpretation. These findings align with the work of Getenet and Callingham (2017), who discovered that understanding fractions necessitates comprehending the fraction as an interconnected concept, specifically as part-whole, ratio, operator, quotient, and measure.

The qualitative results furthermore reveal that concrete objects made the addition of fractions meaningful and made it easier to work with operations. The findings concur with Africa et al. (2020) who found that manipulating concrete objects during the use of the CPA approach enables learners to carry out addition and subtraction operations of fractions. Additionally, Sumartini and Priatna (2018) have found that thinking capabilities tend to progress from concrete to abstract thinking patterns.

The learner demonstrated the capability to link the area model, the part-whole concept, and the length model or a number line. Furthermore, it is essential to highlight that the number line serves as the measurement construct. The response indicates that diagrams and visuals assist learners in establishing connections. These findings contradict those of Ollesch et al. (2017), who noted that images may induce extraneous cognitive load as such they may not be able to establish connections between mathematics concepts.

CONCLUSION

In this paper, we concluded that the use of the CPA approach significantly enhanced learners' conceptual understanding of fractions. The most notable enhancement was seen in solving word problems involving fractions. The quantitative analysis revealed an improvement in learners' conceptual understanding of fractions in the post-test. However, it is found that the effect of the intervention is minimal and of negligible size. The qualitative findings indicate that the use of the CPA approach enhanced the learners' conceptual understanding of fractions at the comprehension, operations and connections levels. This study therefore contributes to innovative teaching methods of fractions, potentially enhancing mathematics performance and inspiring teachers to explore new teaching approaches. The study recommends the use of concrete materials for enhancing learners' conceptual understanding when teaching fractions in primary schools. Furthermore, this study recommends the exploration of other approaches which may enhance conceptual understanding of fractions. This study also recommends more research on the CPA approach to reduce poor Mathematics performance and contribute to conceptual understanding theory.

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