**Looking Back** dalam Menyelesaikan Masalah Representasi Aljabar pada Siswa

Hesekiel K. Iilonga¹, Ugorji I. Ogbonnaya²*

¹,²University of Pretoria, South Africa

E-mail: iihesekiel@gmail.com¹, ugorji.ogbonnaya@up.ac.za²

**Abstrak**


**Kata Kunci:** looking back, masalah matematika, masalah representasi aljabar, problem-solving

---

**Students' ‘Looking Back’ in Solving Algebraic Word Problems**

**Abstract**

Solving mathematics problems is a complex cognitive process that involves a series of steps. These steps typically include understanding the problem, devising a plan, carrying out the plan, and looking back (checking the solution). While each of these steps is crucial, the ‘looking back’ step has not received much attention in mathematics education research. This study investigated Grade 10 mathematics students’ looking back in solving algebraic word problems in Namibia. The study followed a qualitative approach. The sample of the study was 351 Grade 10 students from ten secondary schools in the Ohangwena Region in Namibia. Data was collected using the Algebra Word problem-solving Achievement Test and Interview. The results show that, in general, the students did not look back on their solutions to the algebraic word problem. While some students indicated that looking back necessitates the consolidation of their work, others said looking back on their solutions could be time-wasting and confusing. These findings point to the need for mathematics teachers in Namibia to incorporate and model the looking-back step of mathematical problem-solving in their mathematics teaching.

**Keywords:** algebra word problem; looking back; mathematical problem; problem-solving
INTRODUCTION

Mathematical problems are often presented in a verbal or written form known as word problems. Word problems are commonly used in mathematics education at various levels, from primary school to advanced university courses. Mathematical word problems are designed to help students apply their mathematical knowledge and skills to real-world or hypothetical situations, making mathematics more meaningful and relevant (Khoshaim, 2020). The word problems often allow for multiple approaches to solving the problems thereby enhancing the students’ problem-solving skills. Word problems can vary in complexity and cover a wide range of mathematical topics, for example, fractions, algebra, probability, geometry, etc.

However, many students find word problems challenging (Powell, Namkung, & Lin, 2022; Verschaffel, Schukajlow, Star, & Van Dooren, 2020) because mathematical word problems require not only mathematical skills, but also the ability to understand the problem, translate words into mathematical symbols, and choose appropriate problem-solving strategies (Freeman-Green, O’Brien, Wood, & Hitt, 2015; Jarosz & Jaeger, 2019; Wang, Fuchs, & Fuchs, 2016). In particular, algebraic word problems have been identified as a challenge for many students in many countries (Bush & Karp, 2013; Jupri & Drijvers, 2016). Solving algebraic word problems has been consistently reported as a significant challenge for many learners in Namibia over the years (Albin & Von Watzdorf, 2019; Directorate of National Examinations and Assessment, 2015, 2016, 2017, 2018; Sikukumwa, 2017).

Solving mathematical problems in general, is a complex cognitive process that involves a series of steps. These steps typically involve understanding the problem, devising a plan, carrying out the plan, and checking the solution (Polya, 1973). While each of these steps is crucial for successful problem-solving in mathematics, the "looking back" step is not often given as much attention as the first three steps, in mathematics education research. Possibly, because the first three phases seem to be self-evident and are a common part of the solving process in mathematics lessons, the fourth one is often omitted (Csachová, 2021). Most students do not bother to ‘look back’ on their problem-solving processes and solutions because after solving a problem, many students appear to believe they have accomplished their mission and stop further exploration (Cai & Brook, 2006; Lee, 2016).

The looking back step in problem-solving is often seen to occur after the problem solver believes they have found a solution, however, it does not occur only when a complete solution is obtained. It can occur while solving a problem where the problem solver may re-examine the problem solution process again and again to check if everything is on the right track (Lee, 2016). It involves going back to the problem, the steps taken, and the final answer to ensure that the solution is correct and logically sound. It serves several important functions in the problem-solving process, including:

(i) Error detection to identify and correct any errors in the solution. Mathematical errors can occur at any stage of the problem-solving process, from misunderstanding the problem statement to making calculation mistakes. By revisiting each step, the problem solver can detect and rectify these errors thereby arriving at correct solutions. The process of detecting and rectifying errors can help build the problem solver's resilience and a growth mindset towards problem-solving.

(ii) Validation of reasoning. The step of looking back allows the problem solver to validate the reasoning and logic used to arrive at the solution. Thus, it helps the problem solver ensure that the steps taken align with the problem's requirements and that the solution addresses the problem.

(iii) Enhancing conceptual understanding: The looking back step promotes a deeper understanding of mathematical concepts. It encourages the problem solver to think critically about why a particular method was chosen and whether alternative approaches might yield the same or different results. This helps the solver to better grasp the underlying principles and concepts behind a problem's solution.

(iv) Greater insight: Looking back on the problem-solving steps can lead to insights that help generalize problem-solving techniques. The ability to apply these generalized strategies to a broader range of problems is a hallmark of mathematical proficiency. Phuntsho & Dema (2019) emphasized the importance of students returning to the completed answer, reviewing it, and discussing it. This procedure will aid in predicting the techniques to employ in resolving future problems and consequently lead to improved mathematical understanding and growth.
Grade 10 Namibian Students ‘Looking Back’ in Solving Algebraic Word Problems

(v) Boost mathematical problem-solving confidence: Successfully verifying one's solution can boost the problem-solving ability of the problem solver.

A few research studies have investigated the looking back step of students in mathematical problem-solving. In'Am (2014) investigated undergraduate mathematics education students’ implementation of Polya’s problem-solving steps in solving Euclidean geometry problems in Indonesia. The author found that the majority of respondents did not review their solutions to the problem. A similar study by Nurkaeti (2018) with elementary school students in Indonesia revealed that the students had difficulty ‘looking back’ on their answers to the given problems. In a study of 389 high school students' skills in solving the Programme for International Student Assessment (PISA) problem in Tegal Regency, Indonesia, Ariana found the students' ability to look back on their solutions to the problems was very low.

Özdemir & Çelik (2021) used Polya’s Model to examine the problem-solving skills of 71 primary education pre-service teachers in a university in the southeastern Anatolia region of Turkey. The researchers found that the pre-service teachers showed the least performance on evaluating [checking] the result concerning the four Polya’s step model of solving a problem. A similar finding was made by Fitriani, Hayati, Sugeng, Srimuliatni, & Herman (2022) in a study of Junior high school students’ problem-solving ability based on the Polya stages in North Sumatra, Indonesia. The authors concluded that “students still have difficulties in solving mathematical problems at the stage of looking back”.

In a recent study, Makwakwa, Mogari, & Ogbonnaya (2024) investigated first-year undergraduate statistics students’ statistical problem-solving skills at a university in South Africa. On the step of looking back [present your answer], the authors observed that most of the participants in the study performed poorly on the step of ‘looking back’ in statistical problem-solving.

Nursyahidah, Saputro, & Rubowo (2018) conducted descriptive qualitative research to profile the problem-solving ability of seventh-grade students with high mathematical ability in learning based on realistic mathematics supported by ethno-mathematics at a public school in Semarang Regency, Central Java, Indonesia. The students’ problem-solving ability was evaluated using Polya’s Four Problem-Solving Steps. On the step of looking back, the result showed that the students were excellent at ‘look back’ as this allowed them to reflect on the problem-solving process and solutions.

Despite the crucial role of the ‘looking back’ step in the mathematical problem-solving process, there has been limited research on students’ application of this step in Namibia. Consequently, the objective of this study was to investigate the ability of Grade 10 Namibian students in the Ohangwena region to "look back" on their solutions to algebraic word problems. The research question addressed was: How proficient are Grade 10 students in the Ohangwena region of Namibia at rechecking their solutions to algebraic word problems?

METHOD

This research employed a qualitative approach and descriptive research design. The population of the study was all Grade 10 students in the Ohangwena region, while the sample of participants was 351 students from ten sampled secondary schools in the Ohangwena region, Namibia. Data was collected using a test algebra word problem test and an interview. All 351 students wrote the test while 20 randomly selected students (two from each school) were interviewed after the test. The test and interview questions were developed by the first author with the guidance of the Namibia Senior Secondary Certificate Grade 10–11 mathematics syllabus. The test comprised six algebraic word problems developed with the guidance of the Namibian JSC Grade 10 mathematics syllabus. There was no time limit on the test to avoid rushing them to a solution but to allow them to show their work and every step they employed. The data for this study was collected after permission to conduct the study was granted from the Ministry of Education, Arts and Cultures and the schools' management, and the students’ informed consent was obtained. The test was administered in the afternoon after normal schooling hours. The students were instructed not to write their names or the names of their schools on the test papers. Instead, they were provided with codes, one representing the learner and the other representing the school. The students were told to take their time to answer the questions and show all the solution steps and rough calculations.

Page 16
After the test was written, 20 students were selected randomly for oral interviews. Two students (a boy and a girl) were selected from each school. The oral interview was used to further explore the students’ engagement in the ‘looking back’ step in solving the test questions. The questions asked at the interview were: “What makes you think that the answers you got from the test were correct? What can you say about rechecking your solution in the test?”

Qualitative content analysis was performed on the students’ solutions from the test and the interview transcripts. To determine if the students rechecked their solution process in solving the algebraic word problem, we checked for indications of ‘looking back’ in their solutions to the questions in their test scripts. The indications of ‘looking back’ were in the form of annotations and comments indicating their thought process or areas they revisited alongside their solutions in their test scripts. The idea was to check if the student ‘looked back’ irrespective of whether the solutions were right or wrong. The interview transcripts were analyzed inductively to check for any evidence of ‘look back’.

RESULTS

The findings from each of the test questions are presented first then followed by the findings from the interview.

The first test question was, “Frans and Meke are friends. Frans took Meke’s mathematics test paper and will not tell her what mark she got. Frans knows that Mike doesn’t like word problems, so he decides to tease her with word problems. Frans says: “I have 2 marks more than you do and the sum of both our marks is equal to 14. What are our marks?”

This problem can be solved using the algebraic method, let Meke’s mark be M, and Frans’s mark be F, since Frans’ mark is greater than Meke’s mark by 2, then $F=M+2$,

Since, $F=M+2$ \hspace{1cm} (1)

and Frans’ and Meke’s marks = 14, then,

$F+M=14$ \hspace{1cm} (2)

Then substitute (1) into (2) to get $(M+2) + M=14$. Hence, $M = 6$ and $F = 8$

After students had obtained that Frans’ mark was 8 while Meke’s mark was 6, they could recheck as follows:

$F+M=14$

$8+6=14$

$14=14$

For this question, there was evidence that 63 (approximately 18%) out of 349 students who attempted the question ‘looked back’ on their solution while 286 students (82%) did not show any evidence of ‘look back’. Among the 63 students who crosschecked their solutions, 18 got the correct answer while 45 did not. Some examples of students’ ‘look back’ on their solutions to question 1 are shown in Figure 1.

![Figure 1: Examples of students’ ‘look back’ on their solutions to question 1.](image-url)
The second test question was, “A father is three times as old as his son, and his daughter is 3 years younger than the son. If the sum of their ages 3 years ago was 63 years, find the present age of each.”

If the son’s current age is x, then the algebraic representation of the problem could be: (x-3)+(3x-3)+(x-6)=63. Solving the equation, x=15.

Crosschecking the answer, (15-3)+(3(15)-3)+(15-6)=63
12+42+9=63
63=63

Very few students; 28 (8%) out of 348 who attempted the question ‘looked back’ on their solutions to the question. This was evidenced by their adding the ages of the father, daughter, and son to see if the sum was equal to 63. Of the 28 students who ‘looked back’ to their solutions, three got the correct answer to the question while 25 did not. Some examples of the work of the students ‘looked back’ in solving question 2 are shown in Figure 2.

Figure 2. Examples of students’ work on rechecking question 2 answers

The third test question was, “A 1-litre bottle of mango juice costs N$2.00 more than a 1-litre bottle of strawberry juice. If 3 bottles of mango juice and 5 bottles of strawberry juice cost N$ 78.00, determine the price of each juice per 1-liter bottle.” The problem could be represented algebraically as follows:

Let Mango be x, and strawberry be y

Then x=y+2
3x+5y=78

Solving the equations gives y = 9 and x =11.

Crosschecking x=9+2 =11 and 3(11)+5(9)=78

Only 29 (8%) of the 348 students who attempted the question ‘looked back’ to crosscheck their answers while most students; 319 (92%) did not ‘look back’. Figure 3 shows examples of students’ rechecking of their solutions to question 3.
The student in Figure 3a who got the correct solution showed and rechecked their work by substituting the price of mango and strawberry juice into the equation. The student in Figure 3b carried out the rechecking but with the incorrect answers.

The fourth test question was, “Natalia thought of a number. She doubled the number, then subtracted 6 from the result and divided the answer by 2. The quotient is 20. What is the number?” If the number is x then the problem could be represented as:

\[(2x - 6)/2 = 20\]. Hence \(x = 23\).

Looking back, \((2(23) - 6)/2 = 20\)

Seventy-seven (22%) of the 349 students who answered the question showed evidence of ‘looking back’ at their solutions. Out of 77 who looked back only 22 students had the correct solution to the question. On the other hand, the majority of students, 272 (78%), did not ‘look back’ at their work.

Figure 4 shows examples of students’ rechecking question 4.

Figure 4 shows that the learner in (a) correctly ‘looked back’, while the learner in (b) also rechecked their work, but the checking was incorrect because \((2(23-6))/2=(2(17))/2=17\), and not 20, as shown by the learner.

The fifth test question was, “In a physics quiz you get 2 points for each correct answer. If a question is not answered or the answer is wrong, 1 point is subtracted from your score. The quiz contains ten questions. Hafo-Letu received 8 points in total. How many questions did Hafo-Letu answer correctly?”

The algebraic representation of the problem could be as follows:

Number of correct and incorrect answers:

\[x + y = 10\]  \hspace{1cm} (1)

Subtracting points for incorrect answers from points for correct answers to get 8 points:

\[2x - y = 8\]  \hspace{1cm} (2)
Adding equations 1 and 2, one gets 3x = 18 or (x = 6). The value of y could be solved from either of the equations 1 or 2 and the solution crosschecked with the other equation.

The finding revealed that only 29 students who attempted the question tried to recheck their work, while a total of 319 (90.9%) did not recheck their solution and three did not attempt to answer the question. Figure 5 shows examples of some students’ attempts to recheck their solutions to question 5.

(a) Rechecked correct solution

(b) Rechecked incorrect solution

Figure 5. Examples of students’ work on ‘looking back’ at question 5 solution.

The student in Figure 5a got the correct solution to the question and ‘looked back’ at the solution while the student in Figure 5b obtained an incorrect answer but also ‘looked back’ at the solution.

The sixth test question was, “On a farm, Tulukeni has goats and chickens. His son counted 70 heads and his daughter counted 200 legs. How many chickens and goats does Tulukeni have?” The following algebraic equations can be obtained from the problem:

If the number of chickens is x and the number of goats is y

For heads: \( x + y = 70 \) - (1)

For the legs: \( 2x + 4y = 200 \) - (2)

From equations 1 and 2:

\[ 2(70 - y) + 4y = 200 \] - (3)

Hence, \( x = 30 \) and \( y = 40 \).

Most of the students in this study who attempted the question (321 or 92%) did not ‘look back’ at their solutions to the question. Only 29 students did ‘look back’ at their solutions to the problem. Out of the 29 students, 11 got the correct solution to the problem while 16 did not get the correct solution. Figure 6 shows examples of some students who ‘looked back’ at their solution.

(a) Complete and correct rechecking

(b) Incomplete or wrong rechecking

Figure 6. Examples of students’ work on rechecking question 6.

The learner in Figure 6a got the correct solution to the problem and showed a strong ability to recheck their work. The learner used a table to categorize the information. The learner in Figure 6b also rechecked their work although they got an incorrect answer to the question.

The students’ ‘looking back’ on their solutions to the algebraic word problems are summarised in Table 1.
Table 1. The students ‘look back’ on their solutions to the algebraic word problems (n =351)

<table>
<thead>
<tr>
<th>Question</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘Looked back’ and had the correct solution</td>
<td>18</td>
<td>3</td>
<td>14</td>
<td>22</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>‘Looked back’ but incorrect solution</td>
<td>45</td>
<td>25</td>
<td>15</td>
<td>55</td>
<td>26</td>
<td>18</td>
</tr>
<tr>
<td>Did not ‘look back’</td>
<td>286</td>
<td>320</td>
<td>319</td>
<td>272</td>
<td>319</td>
<td>321</td>
</tr>
<tr>
<td>No answer</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

The descriptive statistics in Table 1 show that the majority of the students did not do the rechecking after they were done with their work. The students' propensity to ‘look back’ on their solutions to the algebraic word problems is very poor. This suggests that the students do not regard the step of ‘looking back’ in mathematical problem-solving as an important and necessary step of mathematical problem-solving.

To complement the result from the algebraic word problem test, interviews were conducted with 20 randomly selected students. In the interview, the students were asked (i) What made them think that their answers to the test questions were correct? and (ii) What could they say about rechecking their solutions to the test questions?

Only 5 students indicated that they rechecked their solutions after solving the problem and that it enabled them to be sure that the answers were correct. Three said they find motivation in ‘looking back’ at their solution steps and answers in solving the problem. Nine students said that it was a waste of time to recheck their solutions to the questions. Five said that ‘looking back’ confuses them hence they did not ‘look back’ at their solutions after solving the problems. In all, the interviews showed that most of the students do not ‘look back’ at their solution processes and answers to mathematical problem-solving exercises.

**DISCUSSION**

The findings of the study suggest that Grade 10 mathematics students in the Ohangwena region of Namibia lack proficiency in the crucial step of ‘looking back’ when solving algebraic word problems. Most of the students did not engage in this step, indicating a significant gap in their problem-solving strategies. The finding corroborates the findings of many past studies (Arfiana & Wijaya, 2018; Fitriani et al., 2022; In’am, 2014; Makwakwa et al., 2024; Nurkaeti, 2018; Özdemir & Çelik, 2021) that the ‘looking back’ step of mathematical problem-solving is not well engaged by students at different levels of study.

One possible explanation for this lack of proficiency could be an insufficient emphasis on the importance of ‘looking back’ during mathematics instruction. Likely, the students were not used to engaging in the ‘look back’ step in mathematical problem-solving exercises in the classroom. Tjoe (2019) indicated that the ‘looking back’ step was a familiar practice in mathematical problem-solving among high school students in their study. Students often mirror what they observe their teachers do in the classroom. If the teachers do not always check and evaluate their problem solutions in the classroom it is very unlikely that the students will do so. For students to be proficient at all of Polya’s problem-solving steps they need to be sufficiently orientated to solving problems following the steps (Ambrus & Barczi-Veres, 2022; Fitriani et al., 2022). If students are not explicitly taught to review and evaluate their solutions, they may overlook this step altogether. The possible lack of orientation to ‘looking back’ in mathematical problem solving could be the reason why some students indicated in this study said they did not see the need to review their solution processes and answers.

Additionally, the students’ lack of confidence in their mathematical ability could have been the reason for their weak ability at the ‘look back’ step of problem-solving. If a student is not sure of the process, he or she used to solve a problem, the student may not be likely to bother to recheck the process because rechecking the steps and solution may not help the student notice any errors in the process or may lead to ‘confusion’ as some of the students indicated. It takes one who has a conceptual understanding of the mathematical problem to confidently look back at the problem-solving process.
Students lacking conceptual understanding (low-ability students) are unlikely to look back at the problem because of their lack of understanding of the problem (Simatupang, Elvis Napitupulu, & Syahputra, 2019). Hence, the students’ poor ability to ‘look back’ at the problem-solving process in this study could be an indication of poor conceptual understanding of the problems.

One major implication of the finding of this study is that without effectively ‘looking back’ on their problem-solving solutions, students may miss opportunities to identify and correct errors, leading to a perpetuation of misconceptions and a hindrance to their mathematical understanding and growth (Phuntsho & Dema, 2019). To address this issue, interventions and instructional strategies aimed at promoting the habit of ‘looking back’ should be implemented. This could involve explicit teaching of problem-solving strategies, modeling the ‘looking back’ process, providing opportunities for guided practice, and incorporating reflection into assessments. Additionally, fostering a supportive learning environment where students feel comfortable admitting and addressing mistakes is crucial for encouraging the development of effective problem-solving skills.

CONCLUSION

This study investigated Grade 10 mathematics students’ proficiency in ‘looking back’ while solving algebraic word problems in Namibia. The findings show that the students were not proficient in the step of looking back in solving algebraic word problems as most of the students did not ‘look back’ on their solutions to the algebraic word problem. Furthermore, it was found that some of the students did not even see the need to recheck their process of solving the algebraic word problem or the solutions obtained. This makes a case for mathematics teachers to make the ‘looking back’ step of mathematical problem-solving standard practice in their mathematics teaching.

ACKNOWLEDGMENTS

We acknowledge the support of The Ministry of Education, Arts, and Culture, Namibia, and the Directorate of Education Arts, and Culture, Ohangwena region for granting permission for the study to be conducted in the schools. We also acknowledge the support of the principals of the schools and the students who participated in the study.

REFERENCES


Grade 10 Namibian Students ‘Looking Back’ in Solving Algebraic Word Problems


Problems. *Elementary School Journal*, 122(3). [https://doi.org/10.1086/717888](https://doi.org/10.1086/717888)


Tjoe, H. (2019). “Looking Back” to Solve Differently: Familiarity, Fluency, and Flexibility. [https://doi.org/10.1007/978-3-030-10472-6_1](https://doi.org/10.1007/978-3-030-10472-6_1)
