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Eksplorasi Berpikir Aljabar Siswa Kelas V dalam Menyelesaikan Soal Pola Bilangan

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Abstrak

Penelitian ini bertujuan menyelidiki kemampuan berpikir aljabar siswa kelas 5 sekolah dasar dalam menyelesaikan soal dimensi fungsi. Jenis penelitian yang digunakan dalam penelitian ini adalah deskriptif kualitatif. Subjek dalam penelitian ini sebanyak 131 siswa kelas 5 dari 2 sekolah dasar swasta di Surakarta, Jawa tengah, Indonesia. Instrumen pengumpulan data yang digunakan yaitu 5 soal tes berpikir aljabar dimensi fungsi yang diadopsi dari Raslton yang terdiri dari 2 soal pola numerik dan 3 soal pola gambar. Sebelum digunakan, soal terlebih dahulu divalidasi oleh tiga orang ahli pembelajaran matematika sekolah dasar dan diujicoba pada 10 siswa yang tidak termasuk sebagai subjek penelitian. Analisis data dilakukan dengan melakukan analisis dokumen jawaban siswa dalam menyelesaikan soal tes berpikir aljabar. Penelitian ini membatasi pada analisis data 89 siswa yang termasuk kategori kemampuan berpikir aljabar sedang yang diperoleh dari hasil tes. Analisis dokumen difokuskan pada langkah penyelesaian dan strategi yang digunakan untuk menyelesaikan soal. Hasil penelitian menunjukkan bahwa siswa lebih mampu menyelesaikan pola bentuk linier dibandingkan pola nonlinier. Siswa juga lebih mampu menyelesaikan pola numerik dibandingkan pola gambar. Selain itu, strategi yang digunakan untuk menyelesaikan pola numerik yaitu menentukan kelipatan bilangan dan mengidentifikasi selisih antar suku pada pola. Sedangkan pada penyelesaian pola gambar, strategi yang digunakan yaitu menggambar pola dan mengidentifikasi selisih antar suku pada pola. Penelitian ini menyimpulkan bahwa sebagian besar subjek dengan kategori sedang mampu menunjukkan kemampuan berpikir aljabar dalam dimensi fungsi.

Kata Kunci: berpikir aljabar, fungsi matematika, pola bilangan

Exploration of Fifth Grade Students' Algebraic Thinking in Solving Functional Dimension Problems

Abstract

This study aims to investigate the algebraic thinking skills of fifth grade elementary school students in solving functional dimension problems. The type of research is descriptive qualitative. The subjects in this study were 131 grade 5 students from two private elementary schools in Surakarta, Central Java, Indonesia. The data collection instrument used was 5 questions on the dimension function algebraic thinking test adopted from Raslton which consisted of 2 numerical pattern questions and 3 picture pattern questions. Prior to use, the questions were first validated by three elementary school mathematics learning experts and tested on 10 students who were not included as research subjects. This research is limited to data analysis of 89 students who belong to the category of moderate algebraic thinking skills obtained from the test results. Data analysis was carried out by analyzing students' answer documents in solving algebraic thinking test questions. Document analysis is focused on solving steps and strategies used to solve problems. The results showed that students were more able to solve linear patterns than nonlinear patterns. Students are also more able to solve numerical patterns than picture patterns. In addition, the strategy used to solve numerical patterns is to determine multiples of numbers and identify the differences between terms in the pattern. Whereas in solving of picture patterns, the strategy used is to draw patterns and identify differences between terms in patterns. This study concluded that most of the subjects in the moderate category were able to demonstrate algebraic thinking skills in functional dimensions.

Keywords: *algebraic thinking; math function; number pattern*

INTRODUCTION

Algebraic thinking involves analyzing mathematical situations in the form of relations using symbols, letters, or other tools to support students' cognitive abilities in understanding algebra at school (Kieran, 2004). Algebraic thinking can also be defined as a mental activity that comprises several thinking activities, including generalization, abstraction, dynamic thinking, analytical thinking, modeling, and organization (Lew, 2004). Furthermore, algebraic thinking is the ability of students to use their thinking skills to generalize patterns and analyze the relationships between numbers in a similarity (Kamol & Har, 2010). Blanton & Kaput (2011) explain that algebraic thinking involves the generalization of mathematical ideas using symbolic representation and representing functional relationships. Thus, algebraic thinking is a thinking skill for understanding relationships between quantities that can be represented in symbolic form.

Experts have proposed components that can demonstrate students' ability in algebraic thinking. tates that there are three components in algebraic thinking: generalization skills, transformation skills, and global meta-level skills. On the other hand, Kriegler (2007) posits that algebraic thinking can be organized into two main components: the development of mathematical thinking tools and fundamental algebraic concepts. The first component includes three topics: problem-solving skills, representation skills, and quantitative reasoning skills. The second component encompasses basic algebraic ideas representing the content domain where mathematical thinking tools develop. These basic algebraic ideas are explored through three focuses: algebra as general arithmetic, algebra as language, and algebra as a tool for functions and mathematical modeling. Another opinion related to algebraic thinking components was proposed by Ralston (2013) who outlined three algebraic thinking components: arithmetic generalization, modeling, and functions. Arithmetic generalization involves the efficient and accurate use of number operations and the application of properties of number operations. Modeling includes the use of arithmetic operations, understanding the relationship between number operations, understanding the meaning of the equals sign, and understanding the variables or symbols in equations. Then, functions include the use of numeric patterns and picture patterns for pattern expansion. This study refers to the algebraic thinking components developed by Ralston (2013) as a framework for analyzing the algebraic thinking abilities of elementary school students. The use of components from Ralston is considered relevant because the formulation of algebraic thinking components developed is based on investigations on elementary school students

Töman & Gökburun (2022) state that algebraic thinking plays a crucial role in enhancing students' academic success. The right learning strategies and a good understanding of algebraic concepts and symbols can support the development of students' algebraic thinking. Algebraic thinking is also vital for students to be able to use the correct combination of language, algebraic representation, and mathematical justification commonly used in problem-solving (Siemon et al., 2022). Basir, Waluya, Dwijanto, & Isnarto (2022) explain that in mathematics education, the concept of algebra is a generalization of arithmetic, making algebra an essential component in mathematics. Algebra is crucial in life; for example, when a student faces a problem, they not only solve the problem but also learn something new. This problem-solving ability is a tool for mathematical thinking and the examination of basic algebraic ideas (Sibgatullin et al., 2022). With the advancement of 21st-century education, education becomes increasingly important to meet the demands of changing times. Indicators of the ability to innovate and learn mathematics according to 21st-century learning are referred to as the 4C's: critical thinking, creativity, collaboration, and communication. In solving algebraic thinking problems, students use analytical, evaluative, and creative skills, indirectly employing the 4C abilities (Sunardi, Kurniati, Sugiarti, Yudianto, & Nurmaharani, 2018). Nur (2020) states that 21st-century learning demands the application of mathematical literacy skills, which is an individual's ability to solve everyday life problems using their mathematical knowledge

Research on algebra in elementary school students has been conducted by several researcher (Eriksson & Eriksson, 2020; Pinnock, 2021; Pourdavood, McCarthy, & McCafferty, 2020; Venenciano, Yagi, Zenigami, & Dougherty, 2019). Eriksson & Eriksson (2020) in their study on first and second grade elementary school students, concluded that the students were able to analyze the arithmetic structure of positive integers and rational numbers through reflective actions with algebraic thinking skills such as addition, subtraction, and division. Pinnock (2021)in his research on elementary school-

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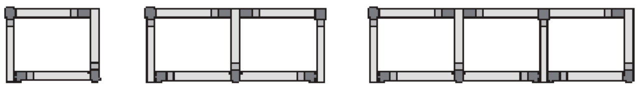
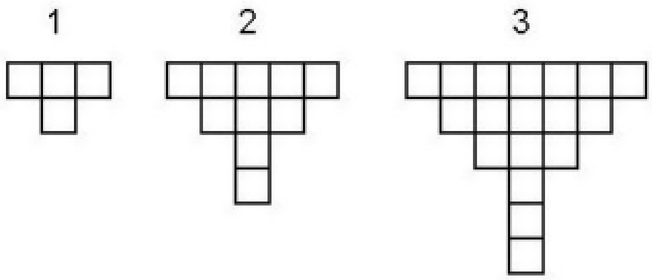
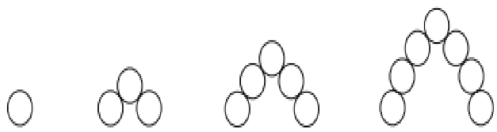
aged students, concluded that the development of students' algebraic thinking could influence algebraic readiness at the middle-school level. Pourdavood et al. (2020) in their study on third and fifth grade elementary students, concluded that participating students who developed an understanding of number patterns and number relationships were able to solve problems using reasoning acquired from mathematical activities and had better communication skills. Venenciano et al. (2019) in their research on first grade elementary students, concluded that the concept of algebraic thinking could develop students' understanding of equation or inequality relationships, such as the equality statement A=B. Based on the findings of these researchers, it can be concluded that algebraic thinking skills are essential for developing understanding of equations and inequalities, problem-solving abilities, enhancing communication skills, and readiness to understand higher-grade algebra.

Though research related to algebraic thinking skills in elementary school students has been conducted by researchers, no study has been found that specifically investigates students' algebraic thinking abilities in function dimensions, including the identification of problem-solving abilities in numeric patterns and picture patterns. Therefore, this study aims to investigate students' algebraic thinking abilities in solving function dimension problems

METHOD

This study is a qualitative descriptive type. The subjects in this study consisted of 131 fifth-grade elementary school students from two private schools in Surakarta, Central Java. The research was conducted in October and November 2022.

Table 1. Algebraic Thinking Test Problems

Problem
1. Determine the next two numbers in the following pattern. 72, 69, 66, 63, 60, …
2. Determine the next two numbers in the following pattern 2, 3, 5, 8, 12, 17, …
3. Look at the following Figures 1 to 3! 
4. Figure 1 is formed from 4 matchsticks, Figure 2 is formed from 7 matchsticks, and Figure 3 is formed from 10 matchsticks. Using the same pattern, determine the number of matchsticks in Figure 5!
5. Look at the following Figures 1 to 3!  Based on the pattern of Figures 1 to 3, determine the number of squares in Figure 5!
6. Look at the following Figures 1 to 4!  Based on the pattern in Figures 1 to 4, determine the number of circles in Figure 10!

The researcher adopted the algebraic thinking components from Ralston (2013), namely numeric manipulation, modeling, and functions, as a framework for analyzing students' algebraic thinking abilities. This study focused on investigating the function dimension of algebraic thinking abilities, including the ability to solve numeric pattern and picture pattern problems. The instrument used was a test of 5 function dimension algebraic thinking problems adapted from Ralston (2013). Before being used, the problems were first validated by 3 elementary school mathematics learning experts. Based on the validation results, the researcher revised the problems according to suggestions regarding the wording to make it easier for students to understand. Subsequently, the researcher conducted a trial of the problems with 10 students who were not part of the study subjects. Based on the trial results, the researcher made further revisions to some problems used for data collection.

The five function dimension problems used for data collection are presented in Table 1. Problem number 1 is used to investigate students' ability to identify and extend linear numeric patterns. Problem number 2 is used to investigate students' ability to identify and extend nonlinear numeric patterns. Problems number 3 and 5 are used to investigate students' ability to identify and extend linear picture patterns. Then, problem number 4 is used to investigate students' ability to identify and extend nonlinear picture patterns.

Next, to analyze the results of the students' algebraic thinking test, the researcher developed an assessment rubric as presented in Table 2.

Table 2. Assessment Rubric

Assessment Criteria Score	Score
Correct solution steps and correct answer	3
Correct solution steps and incorrect answer	2
Partially correct solution steps and incorrect answer	1
Incorrect solution steps or unable to answer the problem.	0

Based on the results of the students' algebraic thinking test, the researcher classified students' abilities into high, medium, and low categories using the score criteria as presented in Table 3. The recapitulation of the number of students in each category is also presented in Table 3.

Table 3. Recapitulation of Student Algebraic Thinking Test Results.

Category	**Criteria Score**	**N**	**Percentage**
High	33,75 – 45	29	22%
Medium	11,25 – 33,75	89	68%
Low	0 – 11,25	13	10%

Based on the data in Table 3, the majority of students, i.e., 89 (68%) students, have medium algebraic thinking ability. In this paper, the researcher analyzes the algebraic thinking ability with a dominant medium category in students. Each subject is given a code of S1, S2, ..., S89 to facilitate data analysis. In this case, the researcher analyzes the steps of problem-solving and the strategies used by students to solve the problems.

RESULT

In this section, the results of document analysis for 89 subjects related to solving function component algebraic thinking problems are presented. The students' algebraic thinking test scores on solving function problems are presented in Table 4.

Table 4. Recapitulation of Students' Algebraic Thinking Test

No	Scores				Total
	Score 0	Score 1	Score 2	Score 3	
1	13	0	39	37	89
2	30	0	31	28	89
3	32	0	1	56	89
4	71	5	1	12	89
5	45	4	1	39	89

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Problem Number 1 Analysis

Problem number 1 concerns the students' ability to identify numerical patterns to extend patterns. Based on the document analysis results, there are 37 (41%) students who are capable of identifying numerical patterns to extend patterns using correct problem-solving steps and answers. Moreover, there are 39 (43%) students able to use numerical relationship patterns to extend patterns using correct problem-solving steps but the answers given are incomplete. Then, there are 13 (14.6%) students who are unable to identify numerical relationship patterns, thus not getting the correct answer.

Figure 1 shows an example of subject S2's answer, who managed to use the available numerical pattern to extend patterns using correct problem-solving steps and answers.

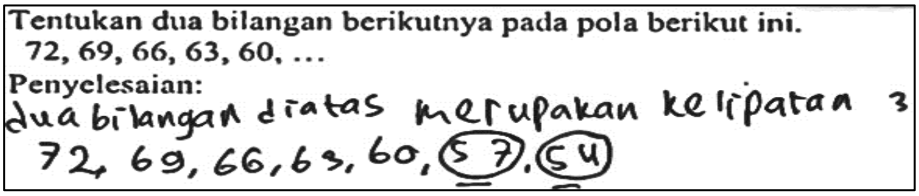

Tentukan dua bilangan berikutnya pada pola berikut ini.
72, 69, 66, 63, 60, ...
Penyelesaian:
dua bilangan diatas merupakan kelipatan 3
72, 69, 66, 63, 60, 57, 54

Figure 1. S2's Answer to Problem Number 1

In Figure 1, S2 writes down the problem-solving steps by determining the pattern form in the number sequence provided in the problem. S2 found that each number in the sequence is a multiple of 3. Then S2 wrote the next two numbers which are 57 and 54. Based on S2's answer to problem number 1, it can be concluded that S2 is capable of identifying numerical patterns to extend patterns.

Figure 2 shows an example of S17's answer, who managed to identify patterns from numerical relationships to extend patterns, but the pattern sought is incomplete. In problem number 1, students are asked to determine the next two numbers from the given pattern, but S17 only wrote one number.

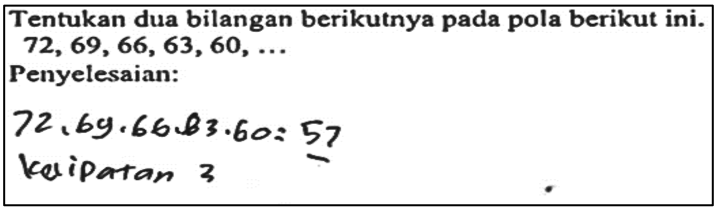
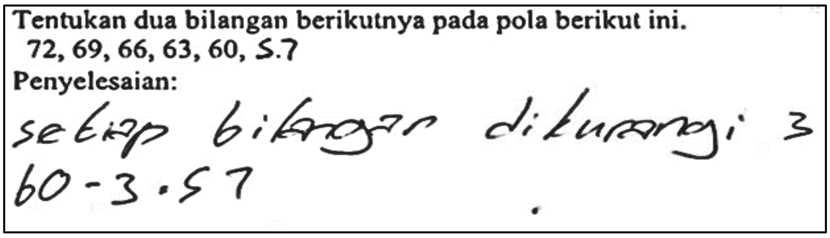

Tentukan dua bilangan berikutnya pada pola berikut ini.
72, 69, 66, 63, 60, ...
Penyelesaian:
72.69.66.63.60: 57
kelipatan 3

Figure 2. S17's Answer to Problem Number 1

In Figure 2, S17 is capable of identifying the next number pattern in the same way as S2, namely that each number is a multiple of 3. Based on the identification result, S17 wrote the answer for the next number, which is 57. However, S17's answer is incomplete because in the problem it is asked to determine the next two numbers that correspond to the numerical pattern given in the problem. Based on S17's answer, it can be concluded that the student is capable of identifying the pattern from numerical relationships to extend the pattern but the answer is incomplete.

Figure 3 also shows an example of S23's answer who is capable of identifying the pattern from numerical relationships to extend the pattern using a different method.


Tentukan dua bilangan berikutnya pada pola berikut ini.
72, 69, 66, 63, 60, 57
Penyelesaian:
setiap bilangan dikurangi 3
60 - 3 = 57

Gambar 3. Jawaban S23 Pada Soal Nomor 1

In Figure 3, S23 is capable of identifying the numerical pattern in the problem, namely the next number is obtained from the operation of the previous number subtracted by 3. Based on the identification result, S23 wrote down the answer of one number after 60 which is 60 subtracted by 3, resulting in 57. Like S17, S23 also only determined one number based on the pattern obtained. S23 did not write down the second number as asked in the problem. Based on S23's answer to problem number 1, it can be concluded that the student is capable of identifying the pattern from numerical relationships to extend the pattern but the answer given is incomplete.

Based on the analysis of student answers to problem number 1, most (more than 80%) of the subjects in the moderate category were able to identify the pattern of numerical relationships to extend the pattern. In addition, there are two strategies in solving problem number 1, namely using multiples of numbers and calculating the difference of sequential numbers in the pattern.

Analysis of Problem Number 2

Problem number 2 is also related to the ability of students to use numerical relationship patterns to extend the pattern. Based on the document analysis results, there are 28 (31%) students who are able to identify the pattern of numerical relationships to extend the pattern using the correct solution steps and the answer obtained is correct. There are also 31 (35%) who are able to identify the pattern of numerical relationships to extend the pattern using the correct solution steps but the answer given is incomplete. In addition, there are 30 (33%) students who have not been able to identify the pattern of numerical relationships to extend the pattern using the correct solution steps so the answer obtained is not correct.

Figure 4 shows an example of S57's answer, who was able to identify the pattern from numerical relationships to extend the pattern, and the answer given was correct.

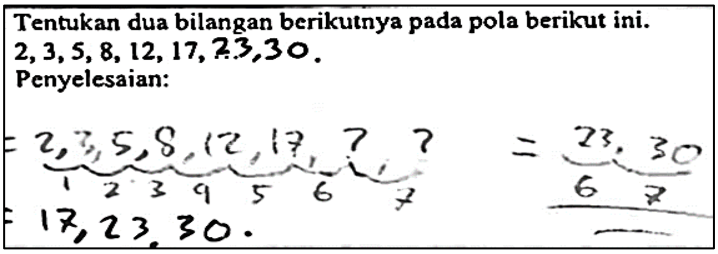


Figure 4. S57's Answer to Problem Number 2

In Figure 4, S57 first identifies the difference between numbers in the pattern presented in the problem. S57 obtained the difference for each number in the pattern, which are 1, 2, 3, 4, and 5. Based on this information, S57 wrote down the difference of the next two numbers as 6 and 7, thus obtaining the next two numbers, which are 23 and 30. Based on S57's answer to problem number 2, it can be concluded that S57 was able to identify the pattern from numerical relationships to extend the pattern using the difference of two numbers in the pattern presented in the problem, and the answer given was correct.

Figure 5 shows an example of S55's answer, who was able to identify the pattern from numerical relationships to extend the pattern, but the pattern obtained was incomplete.

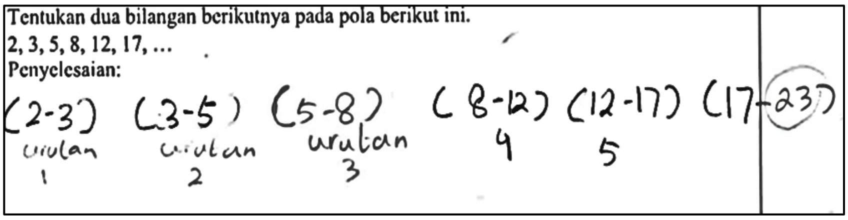


Figure 5. S55's Answer to Problem Number 2

In Figure 5, S55 uses the same method as S57, identifying the difference in each sequence of numbers. Based on the pattern of number differences obtained, S55 can determine the next number, which is 23. However, S55 does not continue the solution step to determine the next number. Based on the result of S55's answer to problem number 2, it can be concluded that S55 is able to identify the

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pattern from numerical relationships to extend the pattern using the difference between two consecutive numbers, but the answer given is incomplete.

Figure 6 shows an example of S61's answer, who was able to identify the pattern from numerical relationships to extend the pattern, but the answer given was less accurate.

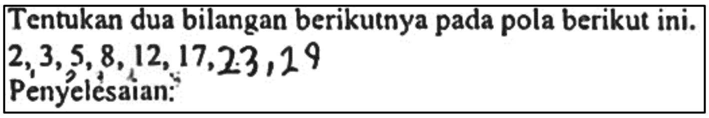


Figure 6. S61's Answer to Problem Number 2

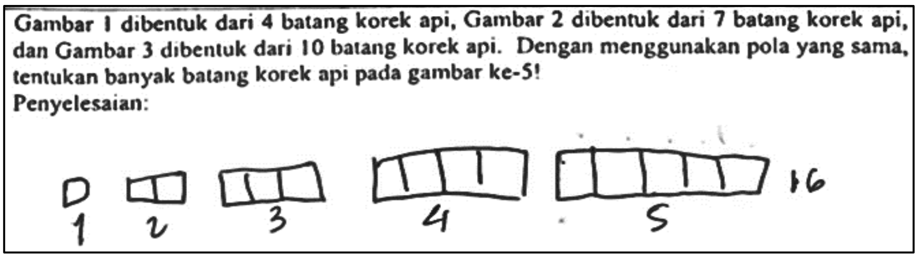
In Figure 6, S61 uses the same method as S55 and S57 by determining the difference in each sequence of numbers presented in the problem. Based on the pattern of number differences obtained, S61 can accurately determine the next number, which is 23. However, S61 made an error in determining the next number where S61 wrote the number sequence after 23 as 29. The correct answer for the number sequence after 23 is 30. Based on S61's answer to problem number 2, it can be concluded that S61 is able to identify the pattern from numerical relationships to extend the pattern using the difference between two consecutive numbers, but the answer given is less accurate.

Based on the analysis of students' answers to problem number 2, most subjects in the moderate category 59 (66%) are able to identify the pattern from numerical relationships to extend the pattern. However, some subjects gave incomplete answers, only writing one number, even though the problem asks for the next two numbers according to the pattern of numbers presented. Also, the strategy used by subjects is relatively the same, that is, using the difference of consecutive numbers in the pattern to determine the next number.

Problem Number 3 Analysis

Problem number 3 is related to the students' ability to use a pattern of images to extend the pattern. Based on document analysis results, there are 56 (62%) students who are able to use the image pattern to extend the pattern using the correct solution steps and obtaining the correct answer. In addition, there are also 32 (35%) students who are not yet able to use the image pattern to extend the pattern using the correct solution steps, so the answer obtained is incorrect.

Figure 7 shows an example of S19's answer, who was able to identify the pattern of images to extend the pattern using the correct solution steps and obtained the correct answer.



Gambar 7. Jawaban S19 Pada Soal Nomor 3

In Figure 7, S19 uses a square image pattern to determine the pattern of the 4th and 5th matchstick arrangements. Based on the data in the problem, in Figure 1 there are 4 matchsticks, in Figure 2 there are 7 matchsticks, and in Figure 3 there are 10 matchsticks. Based on the pattern identification results formed in Figures 1-3, S19 continues to draw the pattern for Figures 4 and 5. Based on the image pattern obtained, S19 determines that Figure 5 is arranged using 16 matchsticks. Based on the results of S19's answer to problem number 3, it can be concluded that S19 is able to identify the image pattern to extend the pattern using the correct steps and the answer obtained is correct.

Figure 8 shows an example of S40's answer, who was able to identify the image pattern to extend the pattern but used a different strategy from S19.

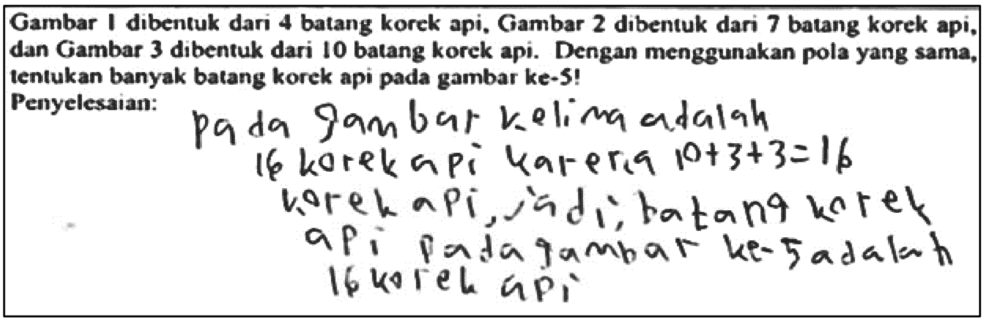

Gambar 1 dibentuk dari 4 batang korek api, Gambar 2 dibentuk dari 7 batang korek api, dan Gambar 3 dibentuk dari 10 batang korek api. Dengan menggunakan pola yang sama, tentukan banyak batang korek api pada gambar ke-5!
Penyelesaian:
pada gambar kelima adalah 16 korek api karena 10+3+3=16 korek api, jadi, batang korek api pada gambar ke-5 adalah 16 korek api

Figure 8. S40's Answer to Problem Number 3

In Figure 8, S40 writes down the solution steps by calculating the difference in the number of matchsticks arranged in each image pattern. S40 identifies that the difference in each matchstick arrangement pattern is 3. Thus, to determine the number of matchsticks in the pattern of Figure 5, S40 adds the number of matchsticks in the pattern of Figure 3 with the difference in the pattern of Figures 3 and 4 and the difference in the pattern of Figures 4 and 5, i.e., 10 + 3 + 3 =16. S40 then concludes that the number of matchsticks in the pattern of Figure 5 is 16 sticks. Based on S40's answer to problem number 3, it can be concluded that the student is able to identify the image pattern to expand the pattern using the difference in the matchsticks in the sequential image pattern.

Figure 9 shows an example of S9's answer, who was not able to identify the image pattern to extend the pattern.

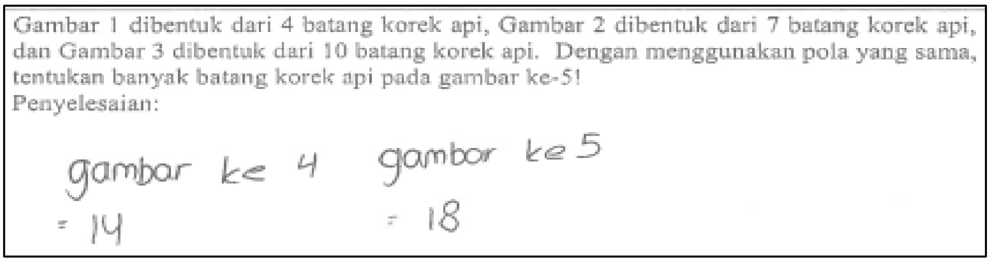

Gambar 1 dibentuk dari 4 batang korek api, Gambar 2 dibentuk dari 7 batang korek api, dan Gambar 3 dibentuk dari 10 batang korek api. Dengan menggunakan pola yang sama, tentukan banyak batang korek api pada gambar ke-5!
Penyelesaian:
gambar ke 4 = 14
gambar ke 5 = 18

Figure 9. S1's Answer to Problem Number 3

In Figure 9, it appears that S1 is not yet able to identify the image pattern to expand the pattern. S1 only wrote down the number of matchsticks in the pattern of Figures 4 and 5, which are 14 and 18. However, the answer given is incorrect. Therefore, it can be concluded that S1 is not yet able to identify the image pattern to expand the pattern.

Based on the analysis of students' answers to problem number 3, most subjects in the moderate category, 56 (62%), were able to identify the numeric relationship pattern to expand the pattern. However, some subjects gave answers that were not correct. In addition, there are two strategies that moderate category subjects used to solve problem number 3. The first strategy uses visual representation, that is, by drawing the next pattern asked. While the second strategy is to find the difference in matchsticks between sequential image patterns.

Analysis of Problem Number 4

Problem number 4 is related to students' ability to identify image patterns to expand the pattern. Based on document analysis results, only 12 (14%) students were able to identify the image pattern to expand the pattern using the appropriate solution steps and the answers obtained were correct. On the other hand, most students, that is 71 (80%), were not yet able to identify the image pattern to expand the pattern.

Figure 10 shows an example of S49's answer, who was able to identify the image pattern to expand the pattern using the correct solution steps and obtained the correct answer.

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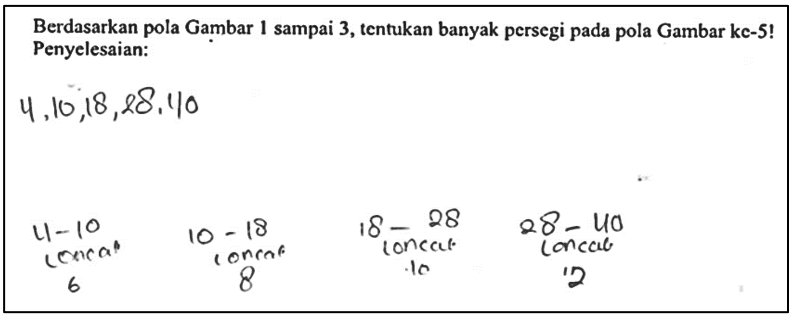


Berdasarkan pola Gambar 1 sampai 3, tentukan banyak persegi pada pola Gambar ke-5!
Penyelesaian:

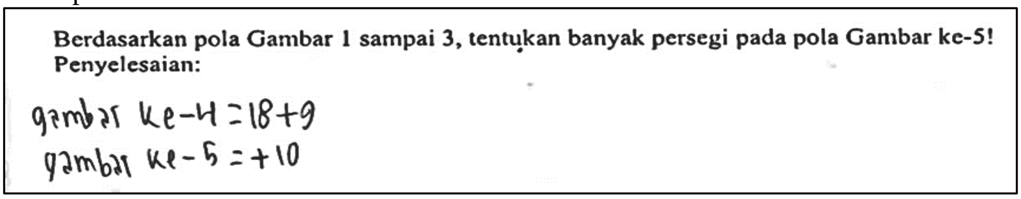
4, 10, 18, 28, 40

4 – 10 loncat 6   10 – 18 loncat 8   18 – 28 loncat 10   28 – 40 loncat 12

Figure 10. S49's Answer to Problem Number 4

In Figure 10, S49 first wrote down the number of squares forming each pattern. S49 identified that the pattern of Figure 1 is formed from 4 squares, the pattern of Figure 2 is formed from 10 squares, and the pattern of Figure 3 is formed from 18 squares. Next, S49 determined the difference in the number of squares in sequential image patterns. The difference in squares of Figure 1 and 2 is 6, and of Figure 2 and 3 is 8. Based on the difference pattern in the number of squares, S49 was able to determine the difference between Figures 3 and 4, and Figures 4 and 5, which are 10 and 12 respectively. Based on the pattern of the difference in the number of squares obtained, S49 could determine the number of squares in Figure 4 as 28, and in Figure 5 as 40. Based on S49's answer to problem number 4, it can be concluded that S49 was able to identify the image pattern to expand the pattern using the correct solution steps and obtained the correct answer.

Figure 11 shows an example of S12's answer, who was not yet able to identify the image pattern to expand the patte



Berdasarkan pola Gambar 1 sampai 3, tentukan banyak persegi pada pola Gambar ke-5!
Penyelesaian:

gambar ke-4 = 18+9
gambar ke-5 = +10

Figure 11. S12's Answer to Problem Number 4

In Figure 11, it appears that S12 directly wrote down the number of squares in the pattern of Figure 4 by adding the number of squares in Figure 3 with 9, that is, 18 + 9. Then S12 wrote for the number of squares in the pattern of Figure 5 by adding the sum of the pattern of Figure 4 with 10. It appears that S12 added the number 9 to the pattern of Figure 4 and 10 to the pattern of Figure 5, which is unrelated to the image pattern in the problem. Based on S12's answer to problem number 4, it can be concluded that S12 was not yet able to accurately identify the image pattern, so the pattern extension given is not correct.

Based on the analysis of student answers, S12 used a strategy to solve problem number 4 by utilizing the difference in the number of squares in sequential square image patterns. The result of S12's identification of the difference pattern in the number of squares from Figure 1 to 3 is used to expand the pattern of the number of squares in Figures 4 and 5.

Analysis of Problem Number 5

Problem number 5 is related to students' ability to use image patterns to expand the pattern. Based on document analysis results, there are 39 (44%) students who were able to identify the image pattern to expand the pattern using the correct solution steps and the answers obtained were correct. Meanwhile, there are 45 (50%) students who have not been able to use the image relationship pattern to expand the pattern using the correct solution steps and correct answer. Figure 12 shows an example of S57's answer, who was able to identify the image pattern to expand the pattern.

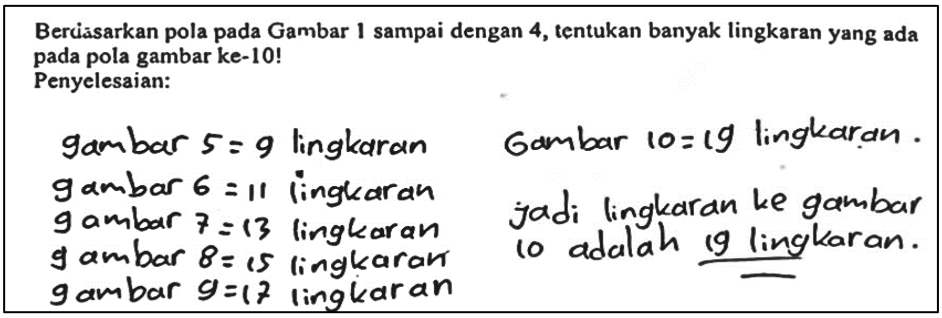

Berdasarkan pola pada Gambar 1 sampai dengan 4, tentukan banyak lingkaran yang ada pada pola gambar ke-10!
Penyelesaian:
gambar 5 = 9 lingkaran
gambar 6 = 11 lingkaran
gambar 7 = 13 lingkaran
gambar 8 = 15 lingkaran
gambar 9 = 17 lingkaran
Gambar 10 = 19 lingkaran.
jadi lingkaran ke gambar 10 adalah 19 lingkaran.

Figure 12. S57's Response to Question Number 5

In Figure 12, S57 utilizes the identified differences between each circular pattern to determine patterns from 5 to 10. S57 identifies the difference between consecutive circular patterns as 2 circles. Based on this information, S57 can determine that pattern 5 contains 9 circles, pattern 6 contains 11 circles, and so on, up to pattern 10, which contains 19 circles. From S57's response to question number 5, it can be concluded that S57 is capable of recognizing the pattern in the images to extend the sequence.

Figure 13 showcases an example of S21's response, which has not yet demonstrated the ability to identify and extend visual patterns.

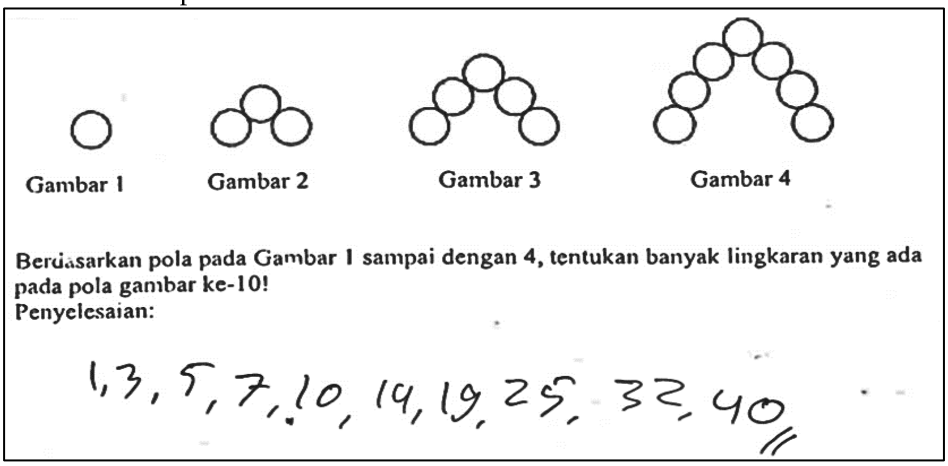


Figure 13. S21's Response to Question Number 5

In Figure 13, S21 lists the number of circles in patterns 1 to 10 as 1, 3, 5, 7, 10, 14, 19, 25, 32, and 40, respectively. However, S21's determination of the number of circles in patterns 5 to 10 is less accurate. This discrepancy is due to S21's errors in identifying the differences in the number of circles between pattern images 4 to 5, 5 to 6, 6 to 7, 7 to 8, 8 to 9, and 9 to 10, which are 3, 4, 5, 6, 7, and 8, respectively. Based on the results of S21's response to question number 5, it can be concluded that the student has not yet mastered the ability to identify and extend visual patterns.

According to the analysis of the students' responses to question number 5, approximately 44% of the subjects have demonstrated the ability to identify and extend visual patterns. However, the majority of subjects fall into the intermediate category (50%), indicating that they have not yet acquired the skill to identify and extend visual patterns accurately. This deficiency stems from errors in identifying the differences between patterns, resulting in inaccuracies in forming subsequent patterns. The strategy employed by the subjects is to seek the differences in the number of circles between patterns to determine the next visual pattern.

DISCUSSION

In the functional dimension, two forms of patterns are analyzed to investigate students' algebraic thinking abilities, namely numeric patterns and visual patterns (Ralston, 2013). In each pattern, the ability to identify and extend linear and nonlinear patterns is examined. Students' ability to identify given

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patterns and extend them to the next pattern serves as an indicator of their algebraic thinking ability. In solving problems related to numeric patterns, the majority of students can identify the relationships and extend numeric patterns. Subjects who can identify and extend linear numeric patterns (question 1) are more numerous than those who can identify and extend nonlinear numeric patterns (question 2), with a percentage of 80% compared to 66%. In other words, subjects in the intermediate category appear to be more capable of solving problems related to linear numeric patterns compared to solving nonlinear numeric patterns. Furthermore, regarding the strategies used to solve numeric patterns, two strategies are observed for solving linear numeric patterns: using multiples of numbers (Figure 1) and identifying the difference between consecutive terms in the pattern (Figures 3 and 5). On the other hand, the strategy for solving nonlinear numeric patterns only involves identifying the difference between consecutive terms in the pattern.

Moving on to solving problems related to visual patterns, subjects who can solve linear visual patterns (questions 3 and 5) appear to be more numerous than those who can solve nonlinear visual patterns (question 4), with percentages of 62% for question 1 and 44% for question 5 compared to 14% for question 4. This indicates that subjects in the intermediate category still struggle in solving problems related to nonlinear visual patterns. As for the strategies used for solving visual patterns, two strategies are observed for solving linear visual patterns: drawing the pattern (Figure 7) and calculating the difference between consecutive terms in the pattern (Figures 8 and 10). The strategy used for solving nonlinear visual patterns involves calculating the difference between consecutive terms in the pattern (Figure 10).

The results of the algebraic thinking test analysis show that, in general, subjects in the intermediate category are more capable of identifying and extending linear patterns compared to nonlinear patterns. In solving nonlinear pattern problems, subjects demonstrate less systematic approaches. Bye, Harsch, & Varma (2022) state that students' strategies in algebraic thinking, especially in complex problems, often lack completeness or include missing operational steps. It is not uncommon for subjects to skip problem-solving steps altogether and directly provide answers. As for the definition of algebraic problem-solving strategies, this involves a systematic problem-solving approach based on rules, where algebraic techniques are applied with insight, and the rules refer to standard rules for strong multiplication and division (Turşucu, Spandaw, & de Vries, 2020). Sugiarti & Retnawati (2019) state that, related to systematic rules, there are several highly varied problem-solving steps.

Some difficulties faced by the subjects in solving problems in the function component include the lack of careful understanding of the questions and the use of incorrect problem-solving strategies. This is in line with Aspari & Anggraini (2018) opinion that implementing algebra using patterns or functions encounters several challenges, ranging from guiding students in viewing patterns as units to generalizing them. During the generalization process, students often make faulty reasoning with incorrect operational concepts or resort to guessing and inappropriate checking (Obara, 2019). The students' test responses indicate that they have not fully mastered some of the materials. One of the contributing factors is the lack of mastery of prerequisite materials and that the materials have not met the learning criteria in certain learning situations (Pratiwi, Farokhah, & Abidin, 2019). The prerequisite knowledge for learning to create patterns in mathematics involves computational concepts, such as addition, substitution, multiplication, and division (Pratiwi, Herman, & Lidinillah, 2018). On the other hand, monotonous teaching strategies and teacher competencies can be categorized as external factors influenced by the students' surrounding environment (Agustyaningrum, Sari, Abadi, & Mahmudi, 2020). Eriksson & Eriksson (2020) state that teaching strategies can be implemented using learning models that demonstrate algebraic thinking. For instance, students are asked to analyze the relationships between quantities, which can also be understood as algebraic thinking. Another learning model that can be employed to enhance algebra learning motivation is by incorporating games with rules that align with algebraic thinking concepts (Sun-Lin & Chiou, 2019).

In the learning process, it is essential to consider strategies that will capture students' interest in algebraic thinking materials. Afonso & Mc Auliffe (2019) state that teaching practices that frequently shift attention from algebraic thinking to arithmetic thinking, such as figural approaches, are more effective than numerical reasoning approaches because they involve recognizing relationships between parts of the figures forming patterns and numerical values. Most students find it easier to solve problems using visuals rather than numerical operations alone. This aligns with research, which indicates that there are two representations that elementary school students find more helpful in problem-solving, namely visual representation and symbolic representation. The development of algebraic thinking strategies can also be applied to increase students' interest and enhance their algebraic thinking abilities. This is supported by Papadopoulos & Patsiala (2019) who show that environmental exploitation, such as through games, can encourage algebraic thinking in students. Demonty, Vlassis, & Fagnant (2018) state that elementary and secondary school teachers can encourage students' progress in developing correct algebraic thinking from elementary to secondary school. As a result, students will perceive the knowledge gained during the learning process as a unified and useful body of knowledge in daily life, thereby sharpening their algebraic thinking skills effectively.

CONCLUSION

The functional dimension is an algebraic thinking component used to assess students' abilities in identifying and extending numeric and visual patterns, consisting of both linear and nonlinear patterns. Subjects in the intermediate category demonstrate better proficiency in identifying and extending linear patterns compared to nonlinear patterns. Additionally, their ability to identify and extend numeric patterns shows a higher percentage compared to their ability to identify and extend visual patterns. The strategies employed by subjects in solving numeric patterns involve using multiples of numbers and identifying the difference between consecutive terms in the numeric patterns. On the other hand, in solving visual patterns, subjects use strategies such as drawing the patterns and calculating the difference between consecutive terms in the visual patterns.

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