

Kesulitan Mahasiswa Calon Guru Matematika dalam Menentukan Basis dan Numerus Materi Logaritma

Surya Kurniawan¹, Suhendra²

^{1,2}Mathematics Education, Universitas Pendidikan Indonesia, Indonesia

E-mail: surya.k@upi.edu¹ suhendra@upi.edu²

Abstrak

Logaritma memiliki peran penting dalam pembelajaran matematika, karena berbagai bidang seperti sains, komputer, keuangan, industri, dan lain-lain menerapkan materi ini sebagai salah satu mata pelajarannya. Khususnya dalam kalkulus, logaritma memiliki peranan penting terutama logaritma natural. Penelitian ini bertujuan untuk mengetahui kesulitan calon guru matematika dalam menyelesaikan permasalahan basis dan numerus pada logaritma. Subjek dalam penelitian ini adalah mahasiswa semester awal di salah satu universitas di kota Banda Aceh yang mengambil mata kuliah aljabar elementer. Penelitian dilakukan dengan pendekatan deskriptif untuk menjelaskan kesulitan yang dihadapi siswa dan menerapkan beberapa langkah seperti memberikan tes logaritma kepada siswa, menentukan kesulitan dan memverifikasi dengan mewawancarai siswa yang mengalami kesulitan berdasarkan jawaban mereka. Hasil penelitian ini menunjukkan bahwa terdapat berbagai kesulitan yang dihadapi mahasiswa yaitu; (1) kurang memahami definisi basis dan numerus logaritma, (2) tidak dapat membedakan ekspresi dan persamaan logaritma, (3) mengingat terlalu banyak rumus sehingga tidak memahami konteks soal, (4) tidak dapat menentukan posisi bilangan irasional pada garis bilangan.

Kata Kunci: kesulitan, logaritma, mahasiswa calon guru matematika

The Difficulties of Pre-Service Mathematics Teacher Students in Determining the BASE and Numerus of Logarithmic Materials

Abstract

Logarithm has an important role in learning mathematics since various fields such as science, computer, finance, industry, and others are applied this material as one of the study subjects. In calculus, logarithm has an important role, especially natural logarithm. This study aims to determine pre-service mathematics teachers' difficulties in solving logarithm bases and numerus material. The subjects are students in the first semester in one university at Banda Aceh city who take elementary algebra course. The research was conducted with a descriptive approach to explain the difficulties that students faced, and it applied several steps such as providing logarithm test to students, determining the difficulties, and verifying it by interviewing students who had difficulties based on their answers. This study revealed that there were various difficulties that students faced such as; (1) lack of understanding of bases and numerus definitions of logarithms, (2) not being able to distinguish between expressions and logarithmic equations, (3) remembering too many formulas so that they do not understand the context of the problem, (4) couldn't determine the position of irrational numbers on the number line.

Keywords: *difficulties; logarithm; pre-service mathematics teacher.*

INTRODUCTION

Mathematics is one of the subjects taught at school, since elementary to college. One of the branches in mathematics is Algebra. A person's success in learning mathematics can be considered by the extent to which students can recognize, justify, provide examples and play a part in this activity (Lumbantoruan & Uly, 2021). Elementary algebra is a branch of mathematics that learn about exponent and logarithm, quadratic equation and function, rational function, and others. All of the studies are important including logarithm since various fields of science need this material such as computers, finance, industry, and others.

Logarithm has some properties, one of that is the basic numerus and basis properties ${}^a\log b = x$, $a > 0$, $a \neq 1$ and $b > 0$. Based on research, students still difficult to solving logarithm problem (Campo-Meneses & García-García, 2020; Dintarini, 2018; Ganesan & Dindyal, 2014b; Glassmeyer, Smith, & Gardner, 2020). The difficulties in learning logarithms in high school students are more focused on the procedural approaches and depended too much on rules rather than the concept of logarithm itself, also student facing difficulty in applying logarithm properties and applying the prerequisite material (Dintarini, 2018; Lima et al., 2019). Similarly, Hurrell (2021) said there is a further chance of students creating conceptual knowledge if they begin through conceptual knowledge and after that move to procedural knowledge. Moving in the opposite way, procedural to conceptual has the risk that students will not work towards conceptual knowledge.

According to Anugrahana (2021), the difficulty experienced while learning mathematics is a weakness in understanding and proving straight objects. Difficulty during recognizing facts, skills, concepts, and principles. Persons who learn mathematics ought to be able to explain the four things that are in the direct object (Lumbantoruan & Uly, 2021). As in logarithm, students must be able to defining logarithm by its concept which is what the base and numerus from logarithm especially if the concept occur in mathematical expression, to find the interval of bases and numerus student should be able to process material of inequality, factorization, the intersection of interval, and others.

Based on preliminary observation in one mathematics education university major in Aceh Province, researchers found that all of the pre-service mathematics teacher students could not answer correctly in logarithm question, in line with (Campo-Meneses & García-García, 2020) that most of mathematics education students have difficulties when solving and understanding laws of exponent and logarithm. This phenomenon indicates that pre-service mathematics teacher students face difficulties when processing their ability to solve the question given. The difficulties might include recognizing facts, skills, concepts, and principles.

Currently, research on logarithms with research subjects is pre-service teacher students still very few, even though this is needed as a consideration that one day the mathematics education students will teach logarithms to high school students. Therefore, researchers think it is necessary to conduct research to identify mathematics students' abilities and the difficulties they face in solving logarithmic problems. This research is expected to be useful for lecturers and students so that they can better anticipate the difficulties faced by students.

METHOD

This research was using descriptive approach. The subjects were pre-service mathematics teacher from one of the University in Banda Aceh, Aceh Province who took elementary algebra course in odd semester year 2020 and who faced difficulties in solving logarithm problem. The data collected by: (1) Test, The test used is the final semester exam in the elementary algebra course which consists of five questions, namely questions about equations and quadratic functions, logarithms, and rational functions. From the results of student answers, it can be seen that there are still many who experience difficulties in solving logarithmic problems, so it is necessary to study more deeply about the difficulties faced by these students. The question is "determine the value of x such as ${}^{2x^2+x-1}\log(3 - 2x)$ has a defined value", (2) Interview, Interview was conducted with the subject who had difficulties based on their answer to gain information about the difficulties to solve logarithm problem that the subject faced. Researchers analyzed the data using Miles and Huberman steps; (1) data reduction, in this step researchers, examined students' answers and grouped their answers based on the difficulties

they might face, after grouping students' answers, an interview was conducted to verify subjects' difficulties, subjects were chosen by purposive from each group. (2) data display, after grouping and interviewing subjects, researchers displayed the finding from this research with figures from subjects' answers, explaining the result from each group by data that were given by subjects. (3) conclusion, this step involves the main finding and suggestions for the next research.

RESULT AND DISCUSSION

Based on the results of tests given to pre-service mathematics teacher students, the difficulties they faced in solving logarithm problems can generally be divided into four types, namely: (1) lack of understanding of bases definitions and numerus of logarithms, (2) not being able to distinguish between logarithm expressions and equations, (3) remembering formulas too much so that they do not understand the context of the problem, (4) difficulty determining the position of irrational numbers on the number line, then from each group was taken as a research subject to be interviewed. From the results of test and interview conducted, the following explanation can be made.

Lack of understanding of bases definitions and numerus of logarithms.

In this study, researcher found that there were still many students who had less knowledge about the base's requirement on the expression $f(x) \log g(x)$ that should be looking is for a solution from $f(x) > 0$ and $f(x) \neq 1$. Figure 1 is the example of student's answer to this type of difficulty.

3. Tentukan batas-batas nilai x agar
 $(2x^2 + x - 1) \log (3 - 2x)$
Jawab
 $2x^2 + x - 1 \log (3 - 2x)$
a $\log b = c$
 $b > 0$
 $3 - 2x > 0$
 $-2x > -3 \quad (-)$
 $2x < 3$
 $x < \frac{3}{2}$
Jadi hp $\{ x \mid x < \frac{3}{2} \}$

Figure 1. Student's 1 Answer

The results of the test and interview concluded that the student wasn't conscientious to draw conclusions about the complete set of x values, so the problem on the question was not answered completely. Subject 1 knew that a is called the base, and b is called numerus and the condition for numerus is a positive number. Even though he knew that a is a basis, student was still confused when asked about the terms of the bases. In contrast, some students have the same difficulties in this type, they know what the requirement of bases but doesn't know how to solve the numerus.

In logarithm the concept that must be understood is the terms bases and numerus, students who do not know the terms of these two things will certainly not be able to solve the existing problems. It was also found by Kusuma (2016), that the students' success in solving problems was influenced more by their conceptual understanding and also similar with Ganesan & Dindyal (2014), Several ideas about logarithms seem not very clear to students, especially the bases and numerus definition. Students' misunderstanding of this logarithm material must be addressed immediately, considering that this student is a prospective mathematics teacher who will teach students about this material, especially at the high school level. If a teacher can not understand logarithms well, it will have an impact on the success of existing learning because research has revealed that the level of mathematics

studied in high school has an influence on the successful performance in college mathematics and science courses (Anderton et al., 2017).

Not being able to distinguish between logarithm expression and equation

Equation is very familiar thing in mathematics, but expression and equation are two different things. In this study, researchers did not give a problem related to solving an equation but a problem in the form of a mathematical expression in logarithms. Figure 2 is an example of a student’s answer to this type of difficulty.

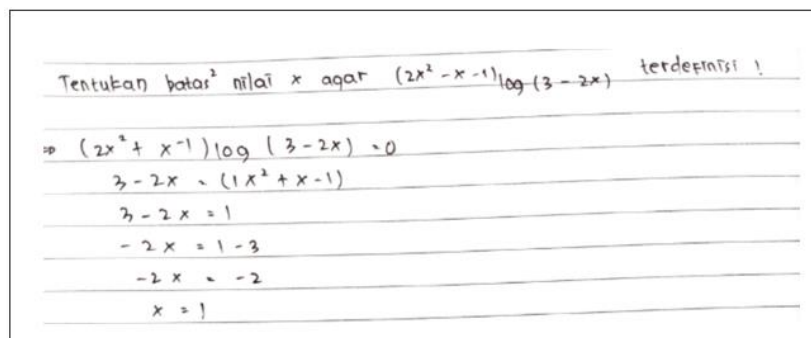


Figure 2. Student’s 2 Answer

Student did not understand the context developed in the question, besides that student also did not give reasons related to what was written on the answer sheet, for example when researchers asked why the equality arose where the question given is only mathematical expression. In addition, the value of $x = 1$ will result in the base of the logarithm expression being 0 so that the logarithm value is not defined.

Student must be able to distinguish between algebraic expression and equation, for instance the $x^2 + x + 2$ is an expression, where $x^2 + x + 2 = 0$ is equation, further $y = x^2 + x + 2$ is a function, expression has no solution, where in equation we can find the solution named root. In this research, student faced difficulties because can’t distinguish between these objects, in contrast, the concept of algebraic expression and equation form the base of teaching higher algebra (Wasserman, 2016), besides that research by YILDIZ et al. (2020) stated that pre-service mathematics teachers have wrong knowledge about algebraic expression that is they defined algebraic expression and equation wrongly , they also found that preservices mathematics teachers still confused when distinguishing mathematics expression and equation.

Remembering too much formulas so they did not understand the context of the problem

Remembering the properties of logarithms is a good thing to understand the essence or evidence of these properties, which is not only fixated on rote memorization but also understand the existing context. Figure 3 is an example of a student’s answer that trying to apply one of the properties of logarithm.

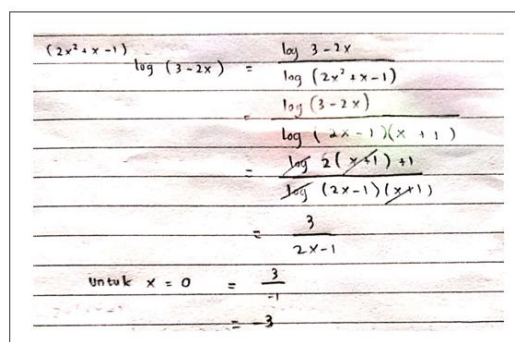


Figure 3. Student’s 3 Answer

Student tried to apply one of logarithm properties, ${}^a\log b = \log b / \log a$, however, when asked what it was used for and what was the use of this formula in this question, student was still confused and could not give a clear reason. When the researchers asked what are the properties of logarithms, student could answer and write them well and correctly, but in the context of problem-solving student was still confused in applying them. The student's answer in figure 3 also showed that the mastery of concepts in the prerequisite material is not good, it could be seen that student did the law of crossing out logarithm numerals without any basis.

Sometimes, we just need to remember mathematics formulas for saving time, but we must also consider that remembering formulas without understanding the proof, or at least the context of them is also a lack of motivation. This research has found students' difficulty when solving logarithm problem because trying to apply logarithm formula, but in actually it is not needed, student can not understand the context of the problem and lack motivation about why he uses the properties of logarithm such as what the purpose, is the properties could bring him to the answer? Similar results were obtained from research conducted by Olkun et al., (2014) in today's education, students memorize the mathematical formulas and therefore obtain difficulty in the solution of the problems requiring formula, and teachers teach based on memorization, without emphasizing the sense and rationale of the formula, also the pre-service teacher is not on the part of making the formula be memorized when teaching the formula to which he/she can provide meaning but makes students straightforwardly memorize the formula to which he/she cannot present a meaning (Koçak & Soylu, 2017). It is advantageous to remember the properties of logarithms as long as we know how they are proved, it is also recommended by Esra & Şükrü (2017) if the teacher involves in the mental process of the rational rule explanations hidden in each subject, understanding would be easier.

Couldn't determine the position of irrational numbers on the number line

Some students had successfully solved the conditions of the base and numerus on the logarithm with the expression ${}^{f(x)}\log g(x)$ that is obtaining the value $f(x) > 0, f(x) \neq 1$ and $g(x) > 0$. However, from the three solutions, the next step is student should determine the intersection of each solution obtained. The solutions that student got in completing $f(x) \neq 1$ form irrational numbers. Figure 4 is an example of a student's answer to this type of difficulty.

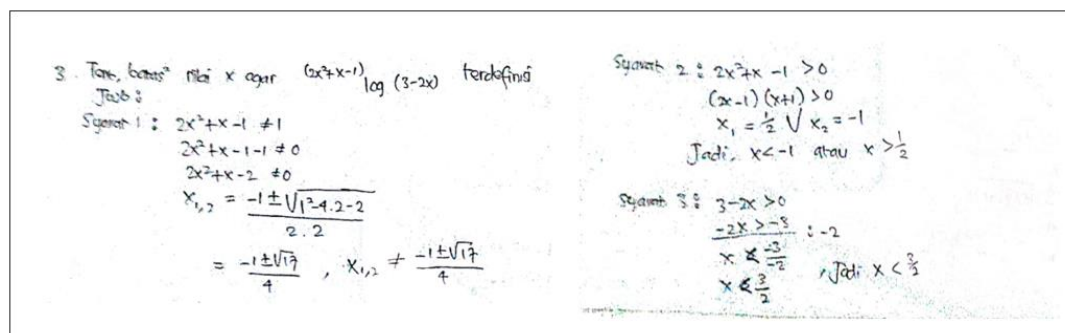


Figure 4. Student's 4 Answer

From figure 4 we can see that student has good abilities, but the student's answer is still incomplete. When researchers asked student how to obtain the fix solution for this problem, he could answer that the fix solution can be obtained by make the intersection among the three solutions that have been obtained, but when the researchers asked the student to determine the position of the numbers on the number line, he was confused to write them down, especially when he tried to write $\frac{-1 \pm \sqrt{17}}{4}$.

The last difficulty experienced by student was the difficulty of placing irrational numbers on the number line, in this study the irrational numbers on the question are $\frac{-1 \pm \sqrt{17}}{4}$. If the student cannot determine the position of the irrational number on the number line, then he will not get a complete answer, because he cannot determine the intersection of several solutions. Similarly, previous research studies have shown that pre-service and in-service mathematics teachers and students contain

difficulty in determining how to place repeating decimals and irrational numbers on the number line (Hayfa & Saikaly, 2016; Kidron, 2018). This is in line with ÇAYLAN ERGENE & ERGENE, (2020) that merely a small number of participants considered that they can be located irrational numbers on the number line. The primary sources of the thought that certain numbers do not have a precise place or cannot be located on the number line were recognized as estimation, infinity, irrationality, and ambiguity in the justifications.

CONCLUSION

From the results of data analysis, student difficulties are caused by four things, namely: 1) lack of understanding of bases definitions and numerus of logarithms, (2) not being able to distinguish between logarithmic expressions and equations, (3) remembering formulas too much so that they do not understand the context of the problem, (4) couldn't determining the position of irrational numbers on the number line. Based on the results of the study, it is necessary to conduct further research on how the difficulties faced by students viewing by their learning motivation

REFERENCES

- Anderton, R., Hine, G., & Joyce, C. (2017). Secondary School Mathematics and Science Matters: Academic Performance for Secondary Students Transitioning into University Allied Health and Science Courses. In *International Journal of Innovation in Science and Mathematics Education* (Vol. 25, Issue 1).
- Anugrahana, A. (n.d.). *Analisis Kemampuan Pemahaman Kognitif Dan Kesulitan Belajar Matematika Konsep "Logika" Dengan Model Pembelajaran Daring Analysis Of Cognitive Understanding Abilities And Difficulties In Learning Logic Mathematics With The Line Learning Model*.
- Campo-Meneses, K. G., & García-García, J. (2020). Exploring the mathematical connections associated with the exponential and logarithmic function in Colombian university students. *Educación Matemática*, 32(3), 209–240. <https://doi.org/10.24844/EM3203.08>
- ÇAYLAN ERGENE, B., & ERGENE, Ö. (2020). Repeating Decimals and Irrational Numbers on the Number Line: Through the Lens of Pre-Service and In-Service Mathematics Teachers. *Acta Didactica Napocensia*, 13(2), 215–232. <https://doi.org/10.24193/adn.13.2.15>
- Dintarini, M. (2018). *Understanding Logarithm: What are the Difficulties That Students Have?* <https://doi.org/10.2991/amca-18.2018.65>
- Esra, A., & Şükrü, İ. (2017). Exploring the opinions about the concepts of formula and rule in mathematics. *Educational Research and Reviews*, 12(19), 956–966. <https://doi.org/10.5897/err2017.3349>
- Ganesan, R., & Dindyal, J. (2014a). *An Investigation of Students' Errors in Logarithms*. MERGA.
- Ganesan, R., & Dindyal, J. (2014b). An Investigation of Students' Errors in Logarithms . (*Proceedings of the 37th Annual Conference of the Mathematics Education Research Group of Australasia*).
- Glassmeyer, D., Smith, A., & Gardner, K. (2020). Developing teacher content understanding by integrating pH and logarithms concepts. *School Science and Mathematics*, 120(3), 165–174. <https://doi.org/10.1111/ssm.12394>
- Hayfa, N., & Saikaly, L. (2016). Dimensions of Knowledge and Ways of Thinking of Irrational Numbers. *ATHENS JOURNAL OF EDUCATION*, 3(2), 137–154. <https://doi.org/10.30958/aje.3-2-3>

- Hurrell, D. (2021). Conceptual Knowledge OR Procedural Knowledge or Conceptual Knowledge AND Procedural Knowledge: Why the Conjunction is Important to Teachers. *Australian Journal of Teacher Education*, 46(2), 57–71. <https://doi.org/10.14221/ajte.2021v46n2.4>
- Kidron, I. (2018). Students' Conceptions of Irrational Numbers. *International Journal of Research in Undergraduate Mathematics Education*, 4(1), 94–118. <https://doi.org/10.1007/s40753-018-0071-z>
- Koçak, M., & Soylu, Y. (2017). Analysis of Pre-service Mathematics Teachers' Teaching Strategy Knowledge of Geometric Formulas. *Universal Journal of Educational Research*, 5(3), 297–315. <https://doi.org/10.13189/ujer.2017.050302>
- Kusuma, H. (2016). How Students Solve The Logarithm? Conceptual and Procedural Understanding. In *Journal of Research and Advances in Mathematics Education ISSN* (Vol. 1, Issue 1). <http://journals.ums.ac.id/index.php/jramathedu>
- Lima, P. da S. N., Silva, L. das A., Felix, I. M., & Brandao, L. de O. (2019). Difficulties in Basic Concepts of Mathematics in Higher Education: A Systematic Review. *2019 IEEE Frontiers in Education Conference (FIE)*, 1–7. <https://doi.org/10.1109/FIE43999.2019.9028658>
- Lumbantoruan, J. H., & Uly, R. (2021). Description of the Difficulties of Mathematics Education Students in Complex Analysis Course. *Journal of Applied Sciences, Mathematics, and Its Education*, 10(1), 2776–3641. <https://doi.org/10.35877/sainsmat716>
- Olkun, S., Çelebi, Ö., Fidan, E., & Engin, Ö. (n.d.). *The Meaning of Unit Square and Area Formula for Turkish Students Göçmen Çocuklar ile Çalışan Öğretmenlerin Matematik Eğitimi Özelinde Mesleki Niteliklerinin Geliştirilmesi View project Mathematics in Nature and Structure Come Alive [TUBITAK 4004] View project*. <https://www.researchgate.net/publication/261436193>
- Wasserman, N. H. (2016). Abstract Algebra for Algebra Teaching: Influencing School Mathematics Instruction. *Canadian Journal of Science, Mathematics and Technology Education*, 16(1), 28–47. <https://doi.org/10.1080/14926156.2015.1093200>
- YILDIZ, P., ATAY, A., & ÇİFTÇİ, S. K. (2020). Preservice Middle School Mathematics Teachers' Definitions of Algebraic Expression and Equation. *International Journal of Contemporary Educational Research*. <https://doi.org/10.33200/ijcer.755359>