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Research Article

DYNAMIC PROCESSES THROUGH THE MATHEMATICAL MODELS AND NUMERICAL EVALUATIONS

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Abstract
This article deals with analytical and numerical methods, particularly focusing on their application in solving displacement problem using mathematical models. This is a study on the evaluation of material damping ratios obtained from experimental data and laboratory tests, an essential criterion to reproduce reality in virtual simulations done for our computational models. The methodology involves iteratively seeking optimal solutions through multiple simulations, providing a way to estimate dynamic responses accurately. The article concludes by emphasizing the importance of numerical-stepping methods in tackling challenging problems in dynamic response analysis. It highlights the significance of numerical-stepping methods for handling non-linear systems and time-varying excitations in dynamic response analysis. This research addresses fundamental issues in the context of applied mechanics. In particular, the article is intended to provide a few computational algorithms where respective graphs are generated, depicting the relationship between time and displacement. Some highly efficient numerical procedures can be developed for linear systems by interpolating the excitation over each time interval and developing the exact solution using a variety of five methods. The numerical results obtained are compared with the theoretical solution. Our research manages to evaluate the depreciation of the material through the models reviewed in the article and gives the results according to the applied models by comparing the results and calculating the absolute error.

Keywords: Approximation, Dynamic, Comparison, Numerical Methods

INTRODUCTION

In this research we use mathematical models in displacement problems associated with the comparison of displacement in theoretical concepts and obtained from simulations and experiments. By exploring these methods, we gain a deeper understanding of their mathematical foundations and practical implications in computational engineering and applied mechanics, marking a significant advancement in the field. Analytical solutions of the equation of motion of a single degree of systems is usually not possible if the excitation applied force of ground acceleration varies arbitrarily with time or if the system is non-linear (Chopra, 2021; Ningsih, 2024). Such problems can be tackled by numerical-stepping methods for integration of differential equations (F. Hoxha, 2017; Pangestu, 2024). A vast body of literature, including major chapters of several books, exists about these methods for solving several types of differential equations that arise in the broad subject area of applied mechanics (Hajrulla et al., 2019; Widodo, 2024).

The literature covers the mathematical formulation of these methods, along with their accuracy, convergence, stability, and implementation in computer software (Merkaj et al., 2021; Fadhilah, 2024). It provides a rich tapestry of approaches designed to simulate physical systems. The five methods selected for comparison in this study have distinct characteristics, advantages, and challenges, as highlighted in various research and applications over the years. The Central Difference method is widely regarded for its simplicity and effectiveness in temporal integration of dynamic equations. Established on finite difference approximations, it offers a direct approach to estimating accelerations and velocities at discrete time steps.

As an extension of the linear interpolation of accelerations between time steps, Linear Acceleration Method enhances the accuracy of dynamic simulations by assuming a linear variation of acceleration. Average Acceleration Method, often associated with the Newmark-Beta family, assumes that acceleration over a time step can be approximated by the average of its values at the beginning and end. Its adaptability to various types of dynamic problems can be highlighted. The Constant Acceleration Method simplifies calculations by assuming constant acceleration within each time step. While this assumption facilitates fast computations, it may compromise the method's applicability to systems with rapidly changing accelerations. The Wilson-Theta Method extends the time step by a factor theta to enhance stability and accuracy in the integration process. It is particularly beneficial for non-linear dynamic analysis, providing a more robust framework for handling large displacements and rotations.

This article, however, only provides a brief overview of a very small number of techniques that are very helpful in dynamic response analysis of single degree of freedom systems (Kapçiu et al., 2024; Khoirunnisa, Triswati, & Coutas, 2024). By interpolating the excitation throughout each time period and generating the exact solution utilizing five methods, several highly efficient numerical algorithms can be created for linear systems. For short time intervals, linear interpolation works well (Li et al., 2020; Apeadido, Opoku-Mensah, & Mensah, 2024). Only a brief presentation of a very few methods that are especially useful in dynamic response analysis of single degree of freedom systems is included here, however (Großeholz et al., 2015; Setiya Rini et al., 2024). Some highly efficient numerical procedures can be developed for linear systems by interpolating the excitation over each time interval and developing the exact solution using five methods. If the time intervals are short, linear interpolation is satisfactory. The numerical results obtained are compared with the theoretical solution. This comparison shows that some numerical methods may predict that the displacement amplitude decays with time, although the system is undamped and that the natural period is elongated or shortened (Bilgin et al., 2023; Bilgin & Ramadani, 2021).

The research is intended to provide only the basic concepts underlying these methods and to provide a few computational algorithms. Although these would suffice for many practical problems and research applications, the reader should recognize that a wealth of knowledge exists on this subject (Li et al., 2020; Tyas & Suttiwan, 2023). A consensus can be underscored that no single method universally stands out across all dynamic analysis scenarios. Each method's performance is inherently linked to the specific characteristics of the problem at hand, including the system's damping, stiffness, and the nature of its dynamic loading. The choice of method, therefore, should be informed by a comprehensive understanding of both the theoretical foundations of these methods and their practical implications in specific applications (Kosovaet al., 2022; Kosova et al., 2023). For linear systems, by interpolating the excitation over each time period and generating the exact solution using five different methods, some very efficient numerical algorithms can be constructed.

Linear interpolation has proven particularly advantageous in cases where computational simplicity is necessary, allowing for efficient computations that are sufficiently accurate for most engineering applications. This is especially useful in scenarios where high-frequency data points are available, as shorter intervals provide a more precise approximation of the excitation forces acting on the system. Additionally, our study includes methods that utilize various approaches to interpolation, offering a range of options that practitioners can select from based on the unique characteristics of their specific applications (Liu et al., 2021; Qodri & Hassan, 2023; Melisa, Nawahdani, & Alam, 2024).

A critical aspect of numerical displacement analysis lies in comparing numerical results to theoretical predictions, which enables a better understanding of each method's accuracy and potential limitations. Through these comparisons, we observe that certain numerical methods may produce discrepancies in displacement amplitude or frequency. For instance, some techniques may incorrectly predict a decay in displacement amplitude over time, even when the system is undamped, or may alter the system's natural period, either elongating or shortening it due to inherent numerical biases (Kapçiu et al., 2016). These deviations underscore the need for practitioners to understand the strengths and limitations of each method, as selecting an unsuitable approach could lead to inaccuracies with significant engineering implications (Almufti et al., 2024; Kapçiu et al., 2024).

In addition to exploring various interpolation techniques, this research also emphasizes the impact of damping on displacement analysis. Damping is a crucial factor in mechanical systems, as it directly influences both the amplitude and frequency of displacement, effectively controlling the system's dynamic response. Accurately modeling damping is essential for creating reliable simulations, as even small changes in damping properties can yield substantially different results. In this study, we utilize experimentally determined damping ratios to enhance the accuracy of our virtual simulations, thereby offering a more realistic representation of system behavior. This approach enables us to validate the performance of each numerical method under conditions that closely mirror real-world dynamics (Kosova et al., 2024; Rini, Oktavia, & Hong, 2024).

Our research methodology involves applying five specific numerical techniques to analyze dynamic displacement in SDOF systems. These methods are carefully selected based on their effectiveness in previous studies and their relevance to dynamic response analysis in applied mechanics. By testing these techniques across a range of damping ratios and excitation patterns, we aim to identify the most reliable methods for specific scenarios, ultimately providing a framework that practitioners can use to select the appropriate approach for their applications (Ahmad et al., 2024; Almufti et al., 2024). Graphical representations of displacement over time serve as an essential component of this research, as they provide a visual comparison of each method's predictions with theoretical expectations. These visualizations offer valuable insights into how each technique manages the effects of damping and time-varying forces, enabling us to clearly see each approach's advantages and drawbacks (Kosova & Sinaj, 2021; Mardiati, Alorgbey, & Zarogi, 2024; Setiyani, Panomram, & Wangdi, 2024). This visual analysis thus plays a crucial role in evaluating the practical relevance of each method, providing engineers with a clearer picture of how each approach may perform in real-world applications.

The objectives of this study are: (1) to evaluate the accuracy, stability, and convergence of each numerical method under different loading conditions and damping scenarios; (2) to provide computational algorithms for each technique, including visualizations of displacement over time, to facilitate practical understanding; and (3) to establish a selection framework that guides method choice based on problem characteristics, such as damping, stiffness, and the nature of dynamic loading. These objectives are designed to address the core issues in dynamic response analysis, providing practitioners with the tools and knowledge necessary to select the most suitable methods for their needs (Hajrulla, L, et al., 2018; Uka et al., 2022; Azis & Clefoto, 2024).

RESEARCH METHOD

It was previously observed that the framework of the tower used high-strength steel with large modulus of elasticity and dynamic load-resistant properties to reduce structural damping, as well as significant vibrational response behaviour (Perez-Martin et al., 2019). Concrete foundations, well known for their massive and stiff nature will establish a low frequency response mode of the tower. Also, we evaluate the energy dissipation characteristics of those surfaces due to damping under a moving load. It provides a unified perspective in the integration of analytic and numerical methods for solving differential equations. In this paper a study is conducted to evaluate material damping ratios extracted from experimental measurements and assumed in laboratory tests considered as an important criterion necessary for the real-life duplication, within virtual computational models.

An SDF system has the following properties: $\mathrm{m} = 0.2533\,\mathrm{kip} - \mathrm{sec}^2/\mathrm{in}, \mathrm{k} = 10\,\mathrm{kips}/\mathrm{in}, \mathrm{Tn} = 1\,\mathrm{s}\,(\omega_\mathrm{n} = 6.283\,\mathrm{rad/sec})$, and $\xi = 0.10$. The response $\mathrm{u(t)}$ of this system to $\mathrm{p(t)}$ with $\Delta\mathrm{t} = 0.1\,\mathrm{sec}$ subjected to half-cycle as shown in Table 1.

Table 1. Given parameters.

Given	Values	Units
m	2.533	$\text{kip} - \text{s}^2/\text{in}$
k	100	kips/in
ξ	0.100	or 10%

Central Difference Method

The central difference method exemplifies one type of this numerical technique used in dynamic analysis to approximate motion in time. It is based on central points of the discretized time domain acceleration and velocity finite difference approximating formulations. The method then iterates to solve for dynamic response by updating the state based off of this approximation at each time step. It is a simple and efficient method but needs to be fine tuned on the time step size that results in stability without high computing times which makes it best suited for moderately non-linear, damped problems (Großeholz et al., 2015).

$$\left[\frac{m}{\Delta t^2}+\frac{c}{2\Delta t}\right]u_{j+1}=p_j-\left[\frac{m}{\Delta t^2}-\frac{c}{2\Delta t}\right]u_{j-1}-\left[k-\frac{2m}{\Delta t^2}\right]u_j \tag{1}$$

where, $\left[\frac{m}{\Delta t^2}+\frac{c}{2\Delta t}\right]=269.215$, $\left[\frac{m}{\Delta t^2}-\frac{c}{2\Delta t}\right]=237.385$ and $\left[k-\frac{2m}{\Delta t^2}\right]=-406.6$.

$$269.215u_{j+1}=p_j-237.385u_{j-1}+406.6u_j \tag{2}$$

$$\dot{u}_j=\frac{u_{j+1}-u_{j-1}}{2\Delta t} \tag{3}$$

$$\dot{u}_j=5\left(u_{j+1}-u_{j-1}\right) \tag{4}$$

Table 2 presents a time-step record of dynamic response parameters for a single system responding to varying force (p(t)). This is a plot of time intervals from 0.0 to 1.0 sec on the following four values: → avg force p(t), position u(t), velocity u̇(t) and acceleration ü(t). Also included is column for theoretical displacement for quick reference.

The system with this applied force is exerted in one time step, and always can be dispended due to the different results of acceleration at each moment. It gives both numerical replies and theoretical values, thus enabling it to compare the computed results with its anticipated answers as illustrated in Fig. 1.

Table 2. Displacement provided from Central Difference Method

Time	p_j	u	$\dot{u}_j$	$\ddot{u}_j$	u (theoretical)
0.0	0.0	0.0000	0.000	0.0000	0
0.1	50.0	0.0000	0.929	18.572	0.0323
0.2	86.6	0.1857	3.011	23.073	0.2254
0.3	100.0	0.6022	4.657	9.853	0.624
0.4	86.6	1.1172	4.379	-15.418	1.0961
0.5	50.0	1.4780	1.578	-40.591	1.4251
0.6	0.0	1.4329	-3.086	-52.690	1.3772
0.7	0.0	0.8609	-6.981	-25.213	0.8683
0.8	0.0	0.0367	-7.822	8.381	0.1105
0.9	0.0	-0.7036	-5.659	34.889	-0.5974
1.0	0.0	-1.0951			-1.0073

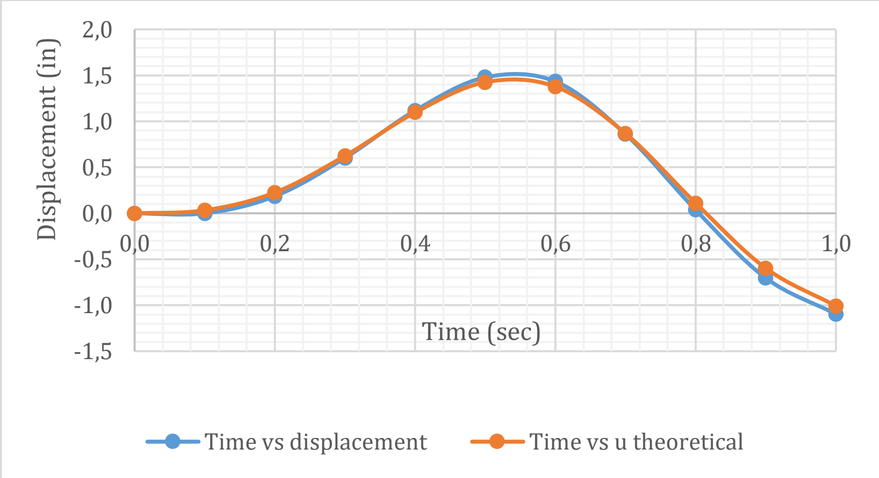

Figure 1. Comparison of theoretical displacement and displacement from Central Difference Method

Newmark's Method and Constant Acceleration Method

This work includes a clear definition of our research targets/subjects (for qualitative) or sample-population (if quantitative). One or the other should also be written down for method in subject selection (qualitative research) and /or sample size technique (Hajrulla, Uka, & Demir, 2023). The family of Newmark methods are a subset among these numerical integration algorithms; used for solving differential equations in structural dynamics. The Newmark techniques are primarily applied to structures of probably dynamic loading, which is treated by seismic force (Hajrulla, Uka, Ali, et al., 2023). The general procedure of Newmark's method has two coefficients gamma and beta, which affect the stability and accuracy thereof (Hajrulla, Bezati, et al., 2018). This can be done in many ways and based of that the method by 3 unique examples like Linear Acceleration Method, Spin boxes or Seekbar, the Average Acceleration Method, and the Constant Acceleration Method (also known as Newmark's Implicit Method) (Perez-Martin et al., 2019; Piña-Flores et al., 2021.

This method, we will refer to as the Constant Acceleration Method and it assumes a constant accleration for an entire time step. Where we have defined the parameters $\gamma = 0.5$ $\beta = 0$. The method is stable and accurate in theory for constant acceleration systems (Mohammadzadeh et al., 2017; Premti et al., 2023). Linear Acceleration Method assumes linear increase/decrease of acceleration over the time step. It is specified using the parameters $\gamma = 0.5$ and $\beta = 1/6$ (average acceleration, average acceleration). This method assumes the acceleration will be constant during time delta across an average of start/end accelerations over each timestep. It is given by $\gamma = 0.5$ and $\beta = 1/4$.

This method is unconditionally stable and it provides a good compromise between accuracy and computational effort (Falcone et al., 2020; Meskouris et al., 2019). Columns for time, displacement (u_j), velocity (u_j)', acceleration (u_j)", the applied force at the next time step ($p_j + 1$), and theoretical displacement facilitate a systematic comparison of numerical results with theoretical predictions at specified intervals, illustrating the method's application in evaluating a system's dynamic response, as shown in Figure 2.

$$\ddot{u}_{j+1} = \frac{1}{m}\left(p_{j+1} - ku_j - (c + k\Delta t)\dot{u}_j - \left(c\Delta t + k\frac{\Delta t^2}{2}\right)\ddot{u}_j\right) \quad (5)$$

where $\left[\frac{1}{m}\right] = 0.395, [c + k\Delta t] = 13.183$ and $\left[c\Delta t + k\frac{\Delta t^2}{2}\right] = 0.8183$

$$\ddot{u}_{j+1} = 0.395\left(p_{j+1} - 100u_j - 13.183\dot{u}_j - 0.8183\ddot{u}_j\right) \quad (6)$$

$$u_{j+1} = u_j + \Delta t\dot{u}_j + \frac{\Delta t^2}{2}\ddot{u}_j \quad (7)$$

$$u_{j+1} = u_j + 0.1\dot{u}_j + 0.005\ddot{u}_j \quad (8)$$

$$\dot{u}_{j+1} = \dot{u}_j + \Delta t \ddot{u}_j \quad (9)$$

$$\dot{u}_{j+1} = \dot{u}_j + 0.1\ddot{u}_j \quad (10)$$

Table 3. Displacement provided from Constant Acceleration Method

Time	p_j	u	$\dot{u}_j$	$\ddot{u}_j$	u (theoretical)
0.0	0.0000	0.000	0.000	50.0	0
0.1	0.0000	0.000	19.739	86.6	0.0323
0.2	0.0987	1.974	27.812	100.0	0.2254
0.3	0.4352	4.755	16.324	86.6	0.624
0.4	0.9923	6.388	-13.012	50.0	1.0961
0.5	1.5660	5.086	-48.475	0.0	1.4251
0.6	1.8322	0.239	-72.634	0.0	1.3772
0.7	1.4929	-7.025	-50.112	0.0	0.8683
0.8	0.5399	-12.036	-6.190	0.0	0.1105
0.9	-0.6946	-12.655	43.326	0.0	-0.5974
1.0	-1.7435	-8.322	79.289		-1.0073

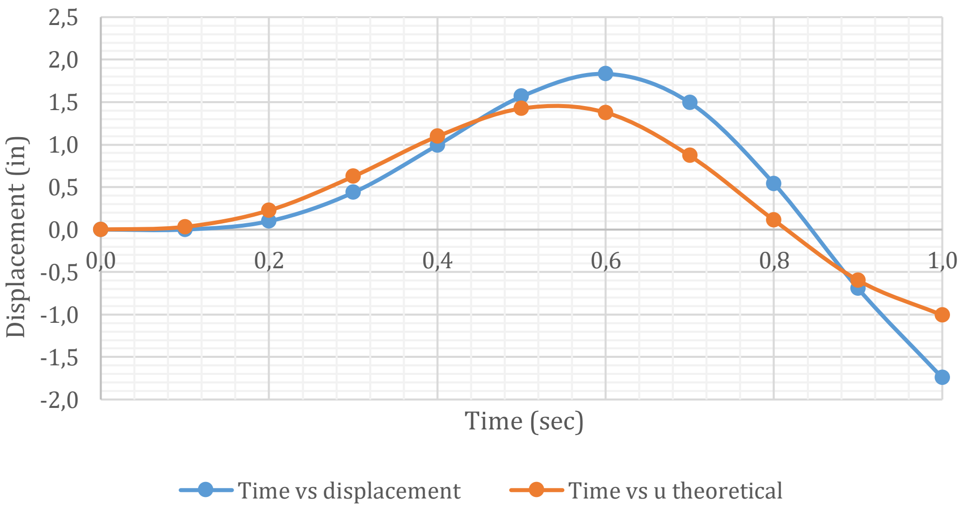

Figure 2. Comparison of theoretical displacement and displacement from Constant Acceleration Method

Linear Acceleration Method and Averrage Acceleration Method

Linear Acceleration Method (Table 4) assumes linearly varying acceleration across the time step. It is defined by the parameters $\gamma = 0.5$ and $\beta = 1/6$. Columns sequentially log time, displacement (u_j), velocity (u_j)', acceleration (u_j)", force at the subsequent time step ($p_j + 1$), and a theoretical displacement, allowing for an evaluation of system behaviour under dynamic (Annastacia et al., 2024). Relate the Average Acceleration Method, the provided code performs several operations to prepare and display a plot comparing computed displacement data with theoretical displacement data as a function of time. Those methods provide a compromise between complexity and computational demand, ideal for cases where acceleration changes moderately over the analysis period and is shown in Figure 3 and Figure 4.

$$\left(\frac{6m}{\Delta t^2} + \frac{3c}{\Delta t} + k\right)u_{j+1} = p_{j+1} + \frac{m}{\Delta t^2}\left(6u_j + 6\dot{u}_j\Delta t + 2\ddot{u}_j\Delta t^2\right) + \frac{c}{\Delta t}\left(3u_j + 2\dot{u}_j\Delta t + \frac{\Delta t^2}{2}\ddot{u}_j\right) \quad (11)$$

Where $\left[\left(6 * \frac{m}{\Delta t^2} + 3 * \frac{c}{\Delta t}\right) u_j\right] = 1{,}615.3$, $\left[\left(6\Delta t * \frac{m}{\Delta t^2} + 2\Delta t * \frac{c}{\Delta t}\right) \dot{u}_j\right] = 158.35$

$$\left[\left(\frac{m}{\Delta t^2}(2\Delta t^2) + \frac{c}{\Delta t}\left(\frac{\Delta t^2}{2}\right)\right)\ddot{u}_j\right] = 5.225 \quad (12)$$

$$\dot{u}_{j+1} = -\frac{\Delta t}{2}\ddot{u}_j - 2\dot{u}_j + \frac{3}{\Delta t}(u_{j+1} - u_j) \quad (13)$$

$$\dot{u}_{j+1} = 30(u_{j+1} - u_j) - 2\dot{u}_j - 0.05\ddot{u}_j \quad (14)$$

$$\ddot{u}_{j+1} = \left(u_{j+1} - u_j - \dot{u}_j\Delta t - \frac{\Delta t^2}{3}\ddot{u}_j\right)\frac{6}{\Delta t^2} \quad (15)$$

$$\ddot{u}_{j+1} = 600(u_{j+1} - u_j) - 60\dot{u}_j - 2\ddot{u}_j \quad (16)$$

Table 4. Displacement provided from Linear Acceleration Method

Time	p_j	u	$\dot{u}_j$	$\ddot{u}_j$	u (theoretical)
0.0	0.0000	0.000	0.000	50.0	0
0.1	0.0291	0.874	17.490	86.6	0.0323
0.2	0.2119	2.860	22.227	100.0	0.2254
0.3	0.5896	4.499	10.547	86.6	0.624
0.4	1.0532	4.382	-12.897	50.0	1.0961
0.5	1.3861	1.870	-37.334	0.0	1.4251
0.6	1.3642	-2.531	-50.678	0.0	1.3772
0.7	0.8967	-6.431	-27.320	0.0	0.8683
0.8	0.1676	-7.647	2.994	0.0	0.1105
0.9	-0.5390	-6.053	28.885	0.0	-0.5974
1.0	-0.9784	-2.519	41.790		-1.0073

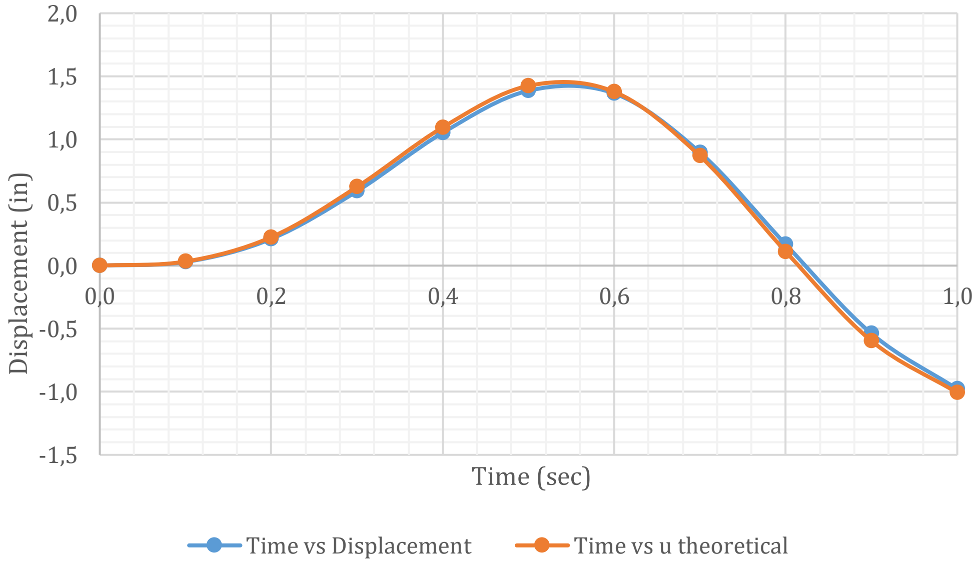

Figure 3. Comparison of theoretical displacement and displacement from Linear Acceleration Method

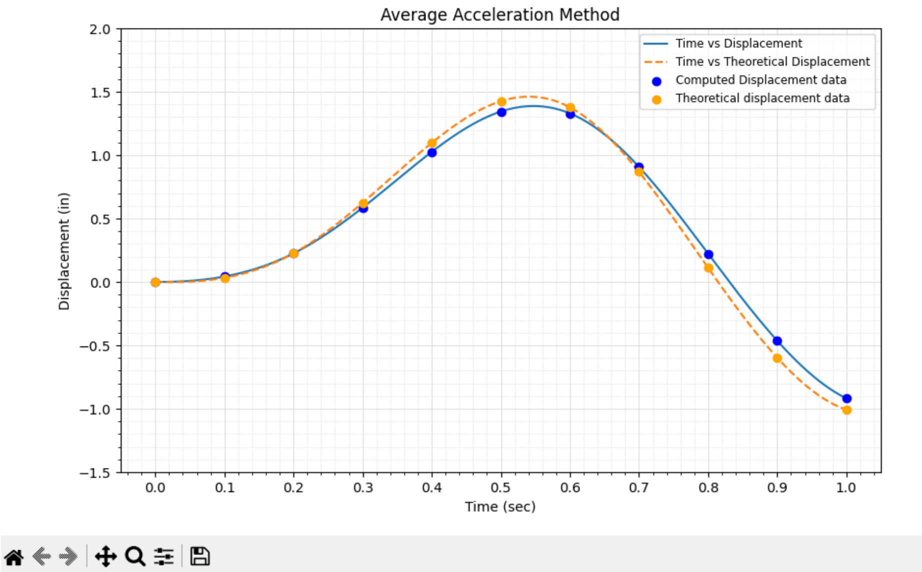

Figure 4. Comparison of theoretical displacement and displacement from Average Acceleration Method

Willson-Theta Method and Central Difference Method

Distinguished by the θ parameter, which typically exceeds 1, this method extrapolates the acceleration over a time interval θ times the actual time step to enhance the stability and accuracy of the response predictions. It is especially beneficial when dealing with larger time steps, as it accounts for the variation in loading over the extended period. The method requires recalculating the equilibrium equations at each θ-incremented time step, using this to update the displacements, velocities, and accelerations in a time-stepped manner, thus allowing for an integrated analysis of a structure's response to dynamic loads.

The summary view of all the curves is shown in Figure 4, highlighting the differences between the methods. The Central Difference and Linear Acceleration methods align most closely with the theoretical curve, indicating higher accuracy. The Wilson-θ Method also shows good agreement, while the Constant and Average Acceleration methods diverge more noticeably, suggesting they are less precise for this scenario.

$$\left(\frac{6m}{(\theta\Delta t)^2}+\frac{3c}{\theta\Delta t}+k\right)u_{j+\theta} = p_{j+\theta} + \frac{m}{(\theta\Delta t)^2}\left(6u_j + 6\dot{u}_j\theta\Delta t + 2\ddot{u}_j(\theta\Delta t)^2\right) + \frac{c}{\theta\Delta t}\left(3u_j + 2\dot{u}_j\theta\Delta t + \frac{(\theta\Delta t)^2}{2}\ddot{u}_j\right) \quad (17)$$

Where $\left[\frac{6m}{(\theta\Delta t)^2}+\frac{3c}{\theta\Delta t}+k\right] = 839.13$, $\left[\left(6 * \frac{m}{(\theta\Delta t)^2} + 3 * \frac{c}{\theta\Delta t}\right) u_j\right] = 739.13$,

$\left[\left(6\theta\Delta t * \frac{m}{(\theta\Delta t)^2} + 2\theta\Delta t * \frac{c}{\theta\Delta t}\right)\dot{u}_j\right] = 107.69$ and $\left[\left(\frac{m}{(\theta\Delta t)^2} * 2(\theta\Delta t)^2 + \frac{c}{\theta\Delta t}\left(\frac{(\theta\Delta t)^2}{2}\right)\right)\ddot{u}_j\right] = 5.305$

$$839.13\, u_{j+\theta} = p_{j+1} + 739.13u_j + 107.69\dot{u}_j + 5.305\ddot{u}_j \quad (18)$$

$$\ddot{u}_{j+\theta} = \left(u_{j+\theta} - u_j - \dot{u}_j\theta\Delta t - \frac{(\theta\Delta t)^2}{3}\ddot{u}_j\right)\frac{6}{(\theta\Delta t)^2} \quad (20)$$

$$\ddot{u}_{j+\theta} = 266.7\left(u_{j+\theta} - u_j\right) - 40\dot{u}_j - 2\ddot{u}_j \quad (21)$$

$$\dot{u}_{j+1} = \dot{u}_j + \frac{\Delta t}{2}(\ddot{u}_j + \ddot{u}_{j+1}) \tag{22}$$

$$\dot{u}_{j+1} = \dot{u}_j + 0.05(\ddot{u}_j + \ddot{u}_{j+1}) \tag{23}$$

Table 5 details the Wilson-θ application in dynamic analysis, providing time-stepped results for displacement(u_j), velocity($\dot{u}_j$), and acceleration ($\ddot{u}_j$) along with applied forces (p_j) and ($p_j + \theta$).

Table 5. Displacement provided from Wilson-Theta Method

Time	u	$\dot{u}_j$	$\ddot{u}_j$	p_j	$p_{j+\theta}$	$u_{j+\theta}$	$\ddot{u}_{j+\theta}$	u (theory)
0.0	0.0000	0.000	0.000	0.0	75.0	0.0894	23.834	0
0.1	0.0265	0.794	15.890	50.0	104.9	0.3507	22.911	0.0323
0.2	0.1932	2.617	20.571	86.6	106.7	0.7633	6.181	0.2254
0.3	0.5418	4.195	10.977	100.0	79.9	1.1802	-19.515	0.624
0.4	0.9823	4.276	-9.351	86.6	31.7	1.3927	-42.914	1.0961
0.5	1.3259	2.222	-31.726	50.0	-25.0	1.2227	-52.952	1.4251
0.6	1.3659	-1.658	-45.877	0.0	0.0	0.7003	-19.412	1.3772
0.7	1.0001	-5.363	-28.234	0.0	0.0	0.0142	8.080	0.8683
0.8	0.3630	-6.976	-4.025	0.0	0.0	-0.6010	30.041	0.1105
0.9	-0.3169	-6.243	18.686	0.0	0.0	-0.9622	40.276	-0.5974
1.0	-0.8238	-3.655	33.079	0.0				-1.0073

Figure 5 shows the summary view of all curves and how methods differ. The Central Difference and Linear Acceleration methods coincide with the theoretical curve most closely—inaccuracies are minimal, denoting statistical accuracy. The Wilson-θ Method works also as good or better than the other methods, and Constant/Average Acceleration clearly a bit further off this time (in comparison to methods).

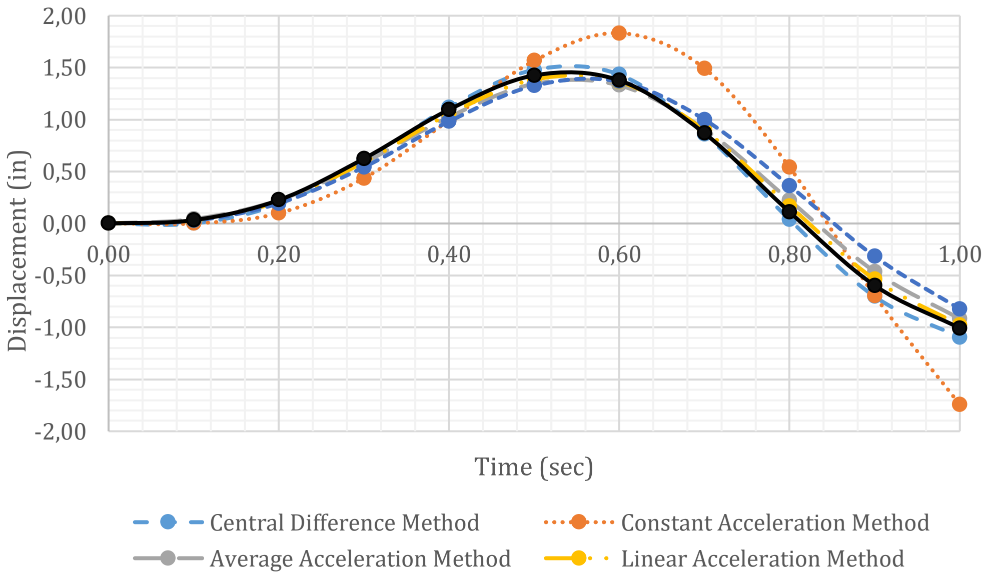

Figure 5. Comparative graph of all methods used

Algorithm Selection

A python algorithm to draw a simulation counter graph is proposed above, which reflects the comparative behavior of accuracy if it deviates when implemented on real case scenario for physical displacement over time. The algorithm will plot two curves, one for time displacement data obtained by numerical analysis method and another one for the theoretical displacement and Matplotlib for plotting (Zhang et al., 2017; Sun et al., 2023; Naimah, Villamor, & Al Wosabi, 2023). The algorithm is going to process the dataset and organize it in structured arrays using libraries like SciPy and NumPy for data organization (Respati et al., 2022; Asmororini, Kinda, & Sen, 2024).

Finally, it will be a plotting function to map out the graph and give each data point its designated mark. The graph plotted in Fig 4, with time on the x-axis and displacement on the y-axis will demonstrate this convergence between numerical predictions to theoretical values. The basic idea is that this visual comparison will help zero in on the precision of our method, by showing where we get curves wrong and right. A close fit indicates a highly accurate numerical method, while significant losses represent opportunities for improvement. This approach makes it possible to give an objective assessment of the effectiveness of a numerical method and correct its parameters in order for them best fit the theoretical model (Sun et al., 2023; Habibi, Jiyane & Ozsen, 2024).

Model Evaluation

This work is based so simulations and results. The code used during the simulation is a python script that plots the displacement as function of time for different numerical methods in a simulation. The concept of the code is to let the user select any type or all successive plotting numerical methods for comperision. For each method a dictionary of displacements data contains with the keys being other and values are database containing displacement value in some time points, calculated from Excel file.

Here the function plot_method is defined to make the plots using user input. It then uses cubic spline interpolation in order to interpolate the displacement data and have smooth curves when plotting it (Bezati et al., 2019). If all methods are selected, the function iterates over each method and plots interpolated displacement curve with scatter points indicating actual atsatenaspoints. The plot is colored and agreeably displayed with different line styles for each of the methods together with a legend at the corner giving labels to all methods. If the user picks certain a method, then for that particular choice, it will plot the interpolated displacement curve along with theoretical. The script outputs the available methods guiding you into choosing before asking for input from a user (Putra & Putra, 2020).

The next part of the code creates an infinite loop that asks the user to select what method they want to plot once plenty of times or exit if it wants. User information is read, then convert to lowercase and use for comparison (so that the user's name matching ignores case) followed by a check of whether it matches with any valid method present in displacements dictionary which we plotted using MatplotLIb or not (Hernández-Montes et al., 2021; Fernande, Sridharan, & Kuandee, 2024; Syahputra & Edwards, 2024).

Assuring the user inputs a valid entry, and then calling plot method function with the corresponding user preferred plotting style to show HeaderInSection in your own Markdown file. If the user enters exit, it exits the look and ending the program. In case of an invalid input, an error message is displayed, prompting the user to provide a valid method name or command. This loop structure ensures an interactive and iterative process, enabling the user to explore different plots of displacement versus time for various numerical methods, as shown in Figure 6.

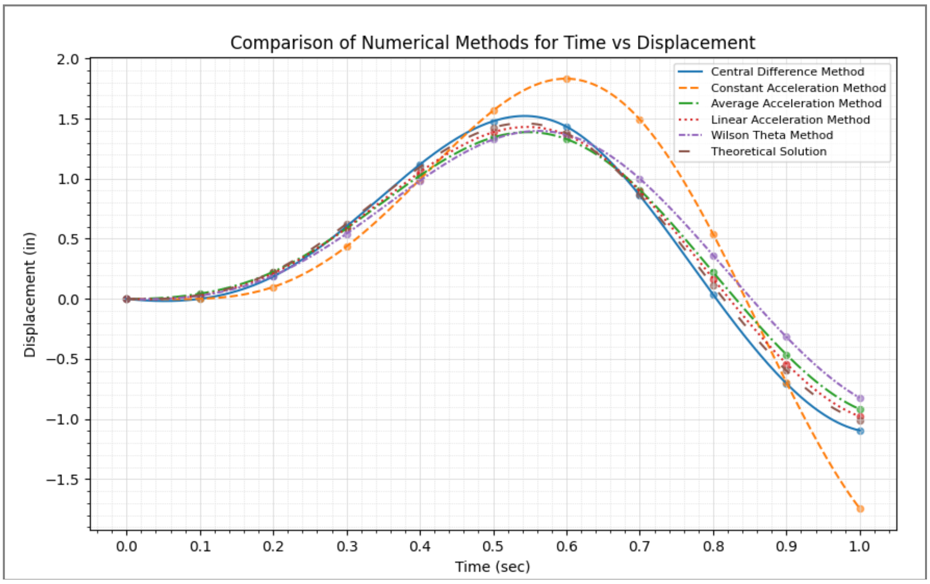

Figure 6. Comparison of theoretical displacement and displacement from each method

Stability and Computer Error

Numerical procedures that lead to bounded solutions if the time step is shorter than some stability limits are called conditionally stable procedures. Procedures that lead to bounded solutions regardless of the time-step length are called unconditionally stable procedures. The Average Acceleration Method is unconditionally stable. The Linear Acceleration Method is stable if $\Delta t/T_n < 0.551$, and the Central Difference method is stable if $\Delta t/T_n < 1/\pi$. Obviously, these two methods are conditionally stable and preferred to be used to calculate dynamic responses.

Any numerical solution to the equation of motion has some error. Five numerical methods are used to address this problem: Wilson-θ method, average acceleration method, linear acceleration method, central difference method, and average acceleration method. The figures produced by applying $\Delta t = 0.1T_n$ are compared with the theoretical solution, which is found by $(t) = \cos\omega_n t$. This comparison shows that some numerical methods may predict that the displacement amplitude decays with time, although the system is undamped, and that the natural period is elongated or shortened.

Four of the methods indicate that the displacement amplitude does not decay. Wilson-θ method does show a decay in amplitude, suggesting that this method introduces numerical damping into the system. The Constant Difference Method introduces the largest period error. In this sense it is the least accurate of the methods considered. For $\Delta t/T_n$ the Central Difference Method and Linear Acceleration Method produce the least period elongation when used in excess of their stability limit. These approaches are better suited for SDF systems among the ones that are offered because of this feature and the fact that there is no amplitude decay.

The choice of time step also depends on the time variation of the dynamic excitation, in addition to the natural vibration period of the system. The time step should also be short enough to minimize distortion of the excitation function. A precise time step is essential for describing numerically the highly irregular earthquake ground acceleration observed during seismic events.

RESULTS AND DISCUSSION

Our research presents the experimental results passing through used numerical methods. We use the comparisons of methods for time and dicplacement. The numerical approaches depicted include the Central Difference Method, Constant Acceleration Method, Average Acceleration Method, Linear Acceleration Method, and Wilson-Method. By experimenting and simulating, getting graphic results and comparing them, we give a very clear idea about the methods used and compare them, analyzing and drawing the appropriate conclusions of our research. Figure 7 shows a comparison between a theoretical solution and various numerical techniques used to approximate the time-displacement relationship of a physical system.

The methods displayed include the Central Difference Method, Constant Acceleration Method, Average Acceleration Method, Linear Acceleration Method, and Wilson-θ Method. Each method produces a curve that oscillates and trends similarly to the theoretical solution, with varying degrees of accuracy The theoretical solution is well captured by the Central Difference Method at all stages, confirming a good accuracy. The Constant Acceleration Method starts instances of divergence that get starker as time goes on. The Average and Linear Acceleration Methods show intermediate performance with slight deviations from the theoretical curve (Dhamo et al., 2024).

The Wilson-θ Method also tracks the theoretical solution closely, although with small differences. All methods seem to converge towards the theoretical solution at the final time point, suggesting that despite the differences in calculation throughout, they may have similar end-point accuracy.

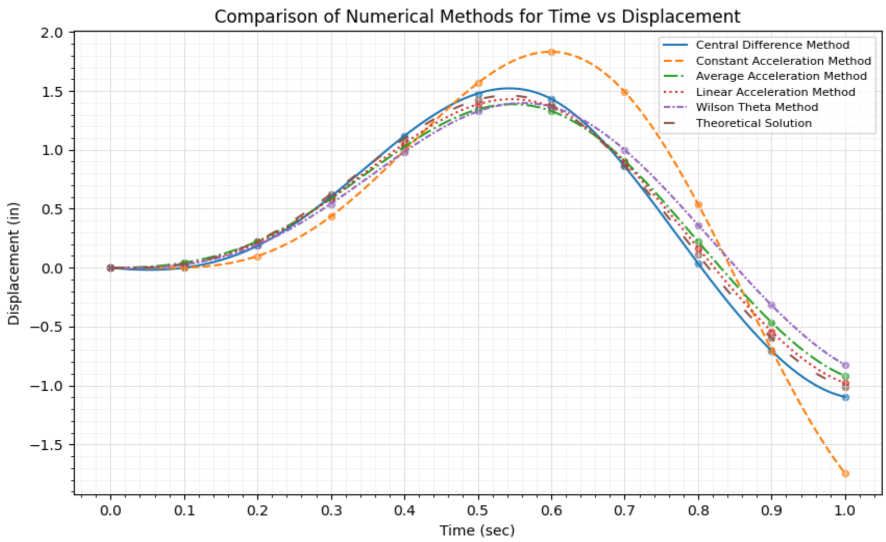

Figure 7. Comparative graph of all methods used, generated in Python

From the analysis used, it appears that the Central Difference Method closely follows the theoretical solution throughout the range, with slight deviations at the peaks and troughs. The Linear Acceleration Method show a similar degree of accuracy to the Central Difference Method, with minor variations that could be due to the simplifications inherent in this method. The Constant and Average Acceleration Methods exhibits a bit more deviation from the theoretical solution, especially in the peaks, suggesting less accuracy in modelling acceleration changes over time. Lastly, the Wilson-θ Method provides a reasonable approximation but with more noticeable deviations than the Central Difference Method, which may be due to its specific assumptions and calculations.

The loop structure ensures an interactive and iterative process, enabling the user to explore different plots of displacement versus time for various numerical methods, as shown in Figure 8.

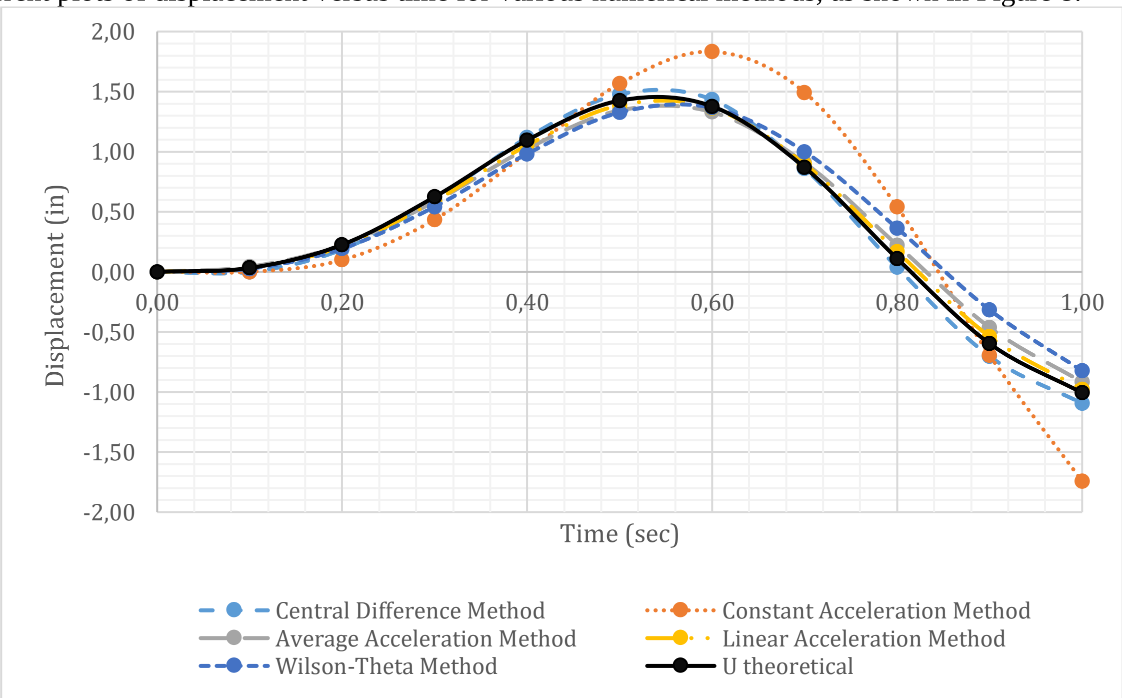

Figure 8. Comparison of theoretical displacement and displacement from methods from Random Generations in Python

In the context of this paper, which researches into numerical procedures for dynamic analysis, methods stand out for their ability to meet the key requirements of convergence, stability, and accuracy. A little above we explained and compared all methods used. The used code defines a procedure for comparing computed and theoretical displacement of an object over time using interpolation to create smooth curves. Arrays of time points and corresponding displacement values are first established; cubic spline interpolation then generates smooth trajectories over a finely spaced time series. In the resulting plot titled “Linear Acceleration Method,” this process visualizes the displacement data (solid line) against the theoretical prediction (dashed line) and marks the original dataset with coloured scatter points blue for computed and orange for theoretical data.

The code provides a flexible way to visualize the displacement versus time for different numerical methods, allowing for easy comparison and analysis. Unique to the Wilson-θ method are the extrapolated force values ($p_j + \theta$) and corresponding response predictions ($u_j + \theta, \dot{u}_j + \theta$) at a future time (θ times the actual time step), which are used to enhance the accuracy of the response calculation. Theoretical displacement serves as a reference for validating the numerical method against expected theoretical values, essential for verifying the reliability of the Wilson-θ method's predictions.

The Central Difference and Linear Acceleration methods align most closely with the theoretical curve, indicating higher accuracy. The Wilson-θ Method also shows good agreement, while the Constant and Average Acceleration methods diverge more noticeably, suggesting they are less precise for this scenario. While each method offers a decent approximation of the theoretical solution, the Central Difference Method and Linear Acceleration Method seems to align most consistently with the theoretical solution, suggesting it may be more reliable for applications where accuracy is principal. Nevertheless, the selection of a method in practice would depend on the specific requirements of the problem at hand, including considerations of computational efficiency and the nature of the data.

To the next future, the final selection of the most suitable numerical method should be guided by the specific context of the dynamic problem, including the nature of the excitation, system properties, and the balance between computational resources and the requisite precision of the results.

CONCLUSION

This study provides a comparative analysis of various numerical methods for calculating displacement over time in dynamic systems, evaluating their accuracy against theoretical solutions. The results demonstrate that the Central Difference Method is a robust and reliable approach, closely approximating theoretical displacement values. The Wilson-θ Method, known for its stability under suitable conditions, also proved effective despite minor deviations from theoretical predictions. The Newmark Method, in its different variations (Linear, Average, and Constant Acceleration), balanced accuracy and computational efficiency, making it a flexible choice for time-step-sensitive problems. Additionally, utilizing Python for numerical simulations enhanced the visualization of results, providing a clearer interpretation of input parameters and assumptions. This highlights the advantage of computational tools in improving the efficiency and accuracy of numerical modeling in structural dynamics. The findings of this study have significant implications for structural analysis and engineering applications. The Central Difference Method and Linear Acceleration Method demonstrated the highest accuracy, making them preferable choices for simulations requiring precision in dynamic response analysis. The results emphasize the need for method selection based on problem-specific requirements, such as stability, accuracy, and computational efficiency. Future research should explore the application of these methods in nonlinear dynamic systems, assess their performance under varying boundary conditions, and integrate machine learning techniques to enhance computational efficiency. Furthermore, developing adaptive numerical techniques that optimize accuracy based on real-time simulation constraints could further improve predictive capabilities in engineering simulations.

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AUTHOR CONTRIBUTIONS

Shkelqim Hajrulla carried out the formulation of the problem, simulations and algorithms, optimization process and run codes. He was responsible for the Statistics. Erind Bedalli has implemented the data results and carried out all conclusions as for the exact results. Robert Kosova has contributed in simulations and algorithms, and optimisation process. Mikaela Cela has implemented the data results, simulations and plotting the graphics.

CONFLICTS OF INTEREST

The author(s) declare no conflict of interest.

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