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Research Article

USING THE MOORE'S THEORY TO EXPLAIN PRESERVICE TEACHERS' DIFFICULTIES IN PROVING OF THE TRIANGLE SUM THEOREM

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Abstract
This study aims to analyze the difficulties of preservice teachers' in proving the triangle sum theorem. The method of this study is used a qualitative method with 58 of preservice mathematics teacher studying for a Bachelor of Education degree in Universitas Negeri Surabaya, Indonesia. The authors analysed the written responses to a 1 item worksheet and also conducted interviews with seven of the participants. The analysis of the data was guided by Moore's theory which was used to identify difficulties of preservice teachers' in proving of the triangle sum theorem. The results showed that still many of preservice teachers still difficulties in proving of the triangle sum theorem. There were 38% of preservice teachers who answered correctly and 62% of preservice teachers answered incorrectly in compiling proof. It was found that several preservice teachers had difficulties in compiling proofs, namely 30 preservice teachers had difficulty understanding the concept, 2 preservice teacher's did not understand the language and mathematical notation and 4 preservice teacher's had difficulty starting the proof. The novelty of this research is introducing a new theoretical analysis related to the difficulty in proving the fundamental theorem of geometry, namely Moore's theory. This study recommends that preservice teacher's should be given solution through scaffolding to help preservice teacher's understand the concept of proof so that students can compiling proofs with correct.

Keywords: Difficulties in Proving, Moore's Theory, The Triangle Sum Theorem, Preservice Teachers.

INTRODUCTION

Proof is at the heart of mathematical thinking and deductive reasoning (Daili, 2022; Balacheff, 2024; Jumaera, Blessing, & Rukondo, 2024; Kalinowski & Pelakh, 2024). A proof is a series of logical arguments that connect true statements to convince the truth of a mathematical statement (Hamami, 2023; Agustiawan & Karti, 2024; Haavold et al, 2024; Mokginta, Suparli, & Mokwena, 2024). In proof, each step is carefully arranged based on established logical rules and principles, so that the final result can provide certainty that the statement to be proven is true. Thus, proof not only serves as a tool to validate mathematical statements, but also to explain and clarify the relationship between various concepts in a mathematical system.

Proof is a series of logical arguments that explain the truth of a statement (Knuth et al, 2019; Hamami, 2022; Arinti, Hamraqulova & Boto, 2024; Trisahid, Kijkosol, & Corrales, 2024). An argument derives its conclusion from the premises of a statement, another theorem, or a definition. Logical means that each step in the argument is justified by previous steps. Hartono et al (2024) explain that proof is a number of logical steps from what is known to reach a conclusion using acceptable inference rules. Hanna (2020) define proof as a collection of true statements linked together logically that serves as a convincing argument for the truth of a mathematical statement. Furthermore, Domingues Stival (2023) provides a definition of proof that involves three different parts, namely (a) to test, try, and determine the actual situation, (b) an argument to convince the expert, and (c) a sequence of formal sentence transformations carried out according to inference rules. From the description above, it can be concluded that proof is a series of logical arguments that convince the truth of a statement.

Apart from being a key element in mathematical thinking, proof also has an important role in education, especially for prospective mathematics teachers (Asamoah et al., 2024; Asrial et al, 2024; Hartono, Siswono, & Ekawati, 2024; Laksono, Suhadi, & Efriani, 2025). Some researchers explain that there are several rules in proof, according to Knuth (2020) and Stylianides et al, (2024) explaining the role of proof as follows: 1) to verify that a statement is true, 2) to explain why the statement is true, 3) to communicate mathematical knowledge, 4) to find or inventing new mathematics, or 5) to systematize statements into axiomatic systems. The purpose of mathematical proof can be stated to prove the truth or falsity of an opinion for each case and condition, as well as showing the relevance of justification. Other purposes of proof such as explanation, systematization, communication, discovery of new results, justification of definitions, developing intuition, providing autonomy (Hanna, 2020; Gunawan, 2023; Aprilia et al., 2024; Kurduka et al., 2024). Planas and Pimm (2024) added that the role of proof is to explain why statements are true and to communicate mathematical knowledge, in particular, supporting the teaching and learning of proof in classrooms where aspects of explanation and communication are valued. Furthermore, Morris (2024) states that the most typical role of proof is to systematize mathematical results into a deductive system of axioms, definitions, and theorems.

Recently, several universities have begun to introduce courses on introduction to proof or mathematical reasoning programs (Leikin et al, 2018; Marasabessy, 2021; Melhuish et al, 2022; Ayuningsih et al., 2024). A strong understanding of the concept of mathematical proof is essential for teachers to ensure that students do not experience difficulties in learning the material. The concept of proof must be given to preservice teachers first at the University, so that after they graduate and work as teachers they can help students in class to understand the concept of proof in geometry. Even though proof is very important, several studies (Siswono et al, 2020; Chin & Fu, 2021; Aisyah et al, 2023; Haj-Yahya et al, 2023; Yolviansyah et al., 2023) show that students still experienced many difficulties in compiling proof. These difficulties can be caused by the lack of concepts possessed by students in studying and compiling proof, did not understand the language and mathematical notation and had difficulty starting the proof. Stylianides et al (2024) research also shows that many students have difficulty understanding the differences between proof, invalid general arguments and empirical arguments. Proof is a complex mathematical activity, so it becomes important in learning mathematics (Komatsu & Jones, 2022; Zhang &Chai, 2021; Rini et al., 2023; Yusnidar et al., 2023). Difficulties arise in teaching proofs to students in the classroom (Siswono et al, 2018; Hartono, 2020). As shown by numerous studies by Miyazaki et al (2024) and Säfström et al (2024); students in all aspects of proofs, have poor understanding and difficulty in constructing their proofs.

In a constructivist learning perspective, difficulties naturally form as part of the learning process, and teachers should try to learn more about these so they can better support their students (Perdana, Zakariah, & Alasmari, 2023; Ramadhanti et al, 2023; Ayu et al, 2024; Pan et al, 2024). When learners construct new knowledge, they do so based on what they already know and understand or do not yet understand. Therefore the process of productive knowledge construction includes a developmental process of difficulty because students' conceptions are embedded in their own individual cognitive systems, which are not a single unit of knowledge. When students create continuous difficulties, this may indicate that there is a concept of the material that has not been ingrained in the students. In this research, we see several difficulties that arise when students have not compiled proof (Hurwitz et al, 2022; Angrist et al, 2022; Dessi & Shah, 2023; Fitriah, Akorede & Agyei, 2023). According to several researchers, there are many difficulties that students often experience in compiling proof, mainly: difficulty in generating deductive arguments and invalid inductive arguments (Ling et al, 2024); difficulty in wrong calculations, inability to follow algebra steps and difficulty in using symbols (Elbir & Medine Ozmen, 2024); difficulty knowing how to choose which facts and theorems to apply, and not having an accurate conception of what is meant by a mathematical proof, and difficulty arranging evidence in a structured manner and difficulty in determining arguments (Hardyanti, Lateef, & Abbas, 2023; Sari, Omeiza, & Mwakifuna, 2023; Säfström et al, 2024).

In addition, several other research literatures also indicate the following areas of potential difficulty that students face in learning to perform proofs: (a) perceptions about the nature of proofs (Polya, 2020), (b) logic and methods of proof (Hilbert, 2019), (c) problem solving skills (Siswono et al, 2020), (d) mathematical language (Velleman, 2019) and (e) conceptual understanding (Rittle-Johnson & Siegler, 2022; Syutaridho et al, 2023). Moore (in Siswono et al, 2020) revealed that there are 7 difficulties experienced by students in constructing proofs, namely: (1) students do not know the definition of certain mathematical objects or concepts needed in the proof, (2) students do not understand the concept, (3) students' concept image is not sufficient in reconstructing proofs, (4) students are unable to generalize from several case examples, (5) students do not know how to use existing definitions, (6) students have difficulty in using mathematical notation and language, and (7) students do not know how to start the proof. In addition, Moore (in Siswono et al, 2020) emphasized the importance of understanding concepts, critical thinking skills, and the ability to argue in the proof process. Moore (in Siswono et al, 2020) also considered that students need to build a deep understanding of mathematical concepts before being able to do proofs well. Therefore, researchers want to use Moore's theory in analyzing students' difficulties in proofs.

By using Moore's theory (in Siswono et al, 2020), researchers can identify the difficulties experienced by students in the context of proof including conceptual understanding, namely students may have difficulty understanding the basic concepts needed to prove a theorem. Then critical thinking skills, namely difficulty in analyzing and evaluating arguments, can also be an obstacle (Nehru et al, 2024). Furthermore, in the use of mathematical language, namely students may find it difficult to express their thoughts mathematically. Apart, from that, there are also many students' difficulties in compiling geometric proofs, especially triangles (Hartono et al, 2024). Based on the researcher's experience in teaching geometry courses, some students still answer many proof questions by rewriting the questions and some even do not do them at all and it is also seen that students often skip proof questions and only work on them when the test time is about to end. This is in line with Sanjaya's research (2016) that there is a striking difference in students' abilities in geometric proofs. In addition, Haj-Yahya et al (2023) showed that there were students who were unable to carry out the process of proving geometric theorems.

Based on the description above and the urgency of research related to the difficulties of mathematics education students in proving basic theorems of geometry, it is very important because proving theorems is the core of a deep understanding of mathematics. Students who will become teachers must have good skills in proving, because they are expected to be able to teach the concept to the next generation. The inability to understand and prove theorems can have a negative impact on the effectiveness of mathematics learning, especially in geometric concepts that are very visual and require strong deductive logic skills. Thus, this study aims to explore the difficulties of preservice teachers' in proving the triangle sum theorem.

RESEARCH METHOD

This study employed a qualitative descriptive and exploratory approach to analyze preservice teachers' difficulties in proving the triangle sum theorem. Descriptive exploratory because this research wants to describe data obtained from in-depth exploration of what mathematics education students do, write and say when completing mathematical proof problems. The subjects in this study were 58 first-year preservice teachers consisting of 20 men and 38 women who took geometry courses at Surabaya State University, Indonesia. The subjects in this study were first-year preservice teachers because they often face challenges in adjusting to new, complex mathematical concepts, including proof. Selecting first-year preservice teachers provides important insights into the difficulties faced when first interacting with geometric materials and formal proof methods. By focusing on first-year preservice teachers, researchers can identify specific difficulties that arise in the early stages of learning, which can be the basis for designing more effective educational interventions.

The data collection were obtained based on the results of the last learning test on students who had difficulty constructing mathematical proof in the geometry mathematics course. The test questions consist of 1 question about proof, namely prove that the number of angles in a triangle is equal to 180^0. The triangle sum theorem, also known as the triangle angle sum theorem or angle sum theorem, is a mathematical statement about the three interior angles of a triangle. The theorem states that the sum of the three interior angles of any triangle will always add up to 180 degrees. This is called a theorem because this is something that can be demonstrated to be true for all triangles. The step for proving of this theorem mainly:

Δ ABC with $\overleftrightarrow{AD} \parallel \overline{BC}$

Given:

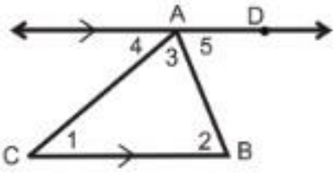


Figure 1. Problem 1

Prove: m $\angle 1$ + m $\angle 2$ + m $\angle 3$ = 180^0

Table 1. Proof of this question

No	Statement	Reason
1	Δ ABC with $\overleftrightarrow{AD} \parallel \overline{BC}$	Given
2	$\angle 1 \cong \angle 4$; $\angle 2 \cong \angle 5$	Alternative interior angles theorem
3	$m\angle 1 = m\angle 4$; $m\angle 2 = m\angle 5$	$\cong$ angles have = measures
4	$m\angle 4 + m\angle CAD = 180^0$	Linear pair postulate
5	$m\angle 3 + m\angle 5 = m\angle CAD$	Angle addition postulate
6	$m\angle 4 + m\angle 3 + m\angle 5 = 180^0$	Substituon (4) and (5)
7	$m\angle 1 + m\angle 3 + m\angle 2 = 180^0$	Substituon (3) and (6)

This question has previously been tested and validated. The time given to complete the task is a maximum of 20 minutes. During the test, students are accompanied by researchers as supervisors with the aim of making the data collected more valid. Then, the student's answer data is scored as right or wrong. For wrong answers, they are then analyzed and coded using Moore's (in Siswono et al, 2020) classification related to students' difficulties in proving in table 1. Moore's theory is used to explain the difficulties of prospective teachers in proving the triangle sum theorem. According to Moore's theory, there are 7 difficulties students have in compiling proof, namely as follows Table 2.

Table 2. Difficulty of proof in mathematics

Code	Difficulty of proof in mathematics
D1	Students do not know the definition and they cannot state the definition.
D2	Students have little intuitive understanding of the concept
D3	Student concept drawings are not sufficient for proof.
D4	Students cannot or do not want to build and use their own examples.
D5	Students do not know how to use definitions to determine the overall structure of proof.
D6	Students are unable to understand and use language and mathematical notation.
D7	Students do not know how to start a proof

For this study, the difficulties in mathematical proof used by Moore (in Siswono et al, 2020) covering more aspects than Miyazaki et al (2024) and Säfström et al (2024). In addition, the researcher views that the components presented by Moore (in Siswono et al, 2020) tend to be more operational than the mathematical proof difficulty components presented by other experts. Moore (in Siswono et al, 2020) found that difficulties did not always stem from a lack of content knowledge. In some cases, students knew the definition and could explain it informally but could not use the definition to write a proof. Students who proved theorems often stalled when starting the proof, which Moore believed was a symptom of some other problem. The sources of these difficulties included deficiencies in the three aspects of conceptual understanding (definition, image, and usage), lack of knowledge of logic and proof methods, and linguistic and notational barriers. Moore (in Siswono et al, 2020) also found that students focused more on procedures than content. All of these components are very much in line with the objectives of this study.

In addition, from the results of previous studies, students relied more on memorizing proofs than understanding proofs because they did not understand what proofs were or how to write them. This reason underlies the researcher to choose the components of mathematical proof difficulties that have been used by Moore (in Siswono et al, 2020) to be used in this study. This reason underlies the researcher to choose the mathematical proving difficulty component that has been used by Moore (in Siswono et al, 2020) to be used in this study. To explore more clearly and deeper regarding the difficulties experienced by students in proving, the researcher conducted interviews with seven selected subjects (A1, A2, …, A7) representing each code in Moore's (in Siswono et al, 2020) classification in Table 1. This study uses Moore's (in Siswono et al, 2020) classification because Moore's theory emphasizes the importance of a deep understanding of geometric concepts before students can do proofs well. This approach is in line with the needs of the curriculum in Indonesia which focuses on understanding concepts, not just memorizing formulas.

RESULTS AND DISCUSSION

The test results were analyzed based on Moore's (in Siswono et al, 2020) analysis, namely the difficulties experienced by students in constructing mathematical proofs. The results of this mathematical proof test (see Table 3).

Table 3. The result of the question

Problem	Answer							
	Correct	Difficulties						
		D1	D2	D3	D4	D5	D6	D7
Prove that the number of angles in a triangle is equal to 180^0	22	7	4	9	7	3	2	4

In this study, there were 38% of students who successfully answered the statement that the sum of the angles in a triangle is 180° correctly. However, further analysis showed that 62% of students who failed had variations in the difficulties faced by students, which were divided into seven categories. Each category of difficulty reflects different challenges that may affect students' understanding and ability to prove this theorem, including they do not know the definition and they cannot state the definition (7 people). Furthermore, they also have little intuitive understanding of the concept (4 people), they concept images are not sufficient for proof (9 people), and they cannot or do not want to build and use their own examples (7 people). In other side, they do not know how to use definitions to determine the overall structure of proof (3 people), they are unable to understand and use language and mathematical notation (2 people), and they do not know how to start a proof (4 people). The following will show some examples of answers to teacher difficulties in proving of the triangle sum theorem:

D1 Students do not know the definition and they cannot state the definition.

Based on Figure 2, it shows that students do not know the definition of opposite angles. They define opposite angles as angles that are different in two parallel lines and in opposite directions. Even though we know that angles that are different in two parallel lines and in opposite directions are definition of alternate angles. The definition of opposite angles should be angles with opposite sides at an intersection of two lines (Zhang & Zhang, 2023).

Translation:

Make an arbitrary triangle and name each vertex A, B, C. Draw a line parallel to side AB and through C, then name the line DE.

Definition

Opposite angles are angles that lie within two parallel lines and are in opposite directions.

$\angle$ BAC = $\angle$ ACD = x^0 (Opposite angles)

$\angle$ ABC = $\angle$ BCE = y^0 (Opposite angles)

So that $x^0 + y^0 + z^0 = 180^0$

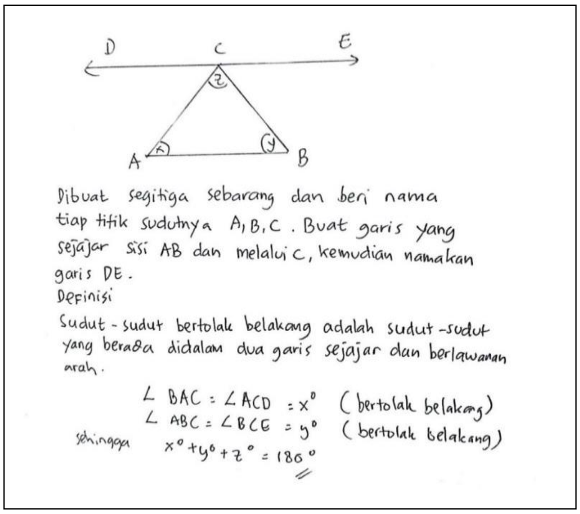


Figure 2. Students do not know the definition and they cannot state the definition

During the interview, the student was probed about his assumptions:

Researcher: is your answer correct?

A1 : Yes, it is correct

Researcher: How did you get this image?

A1 : The figure is obtained by drawing any triangle ABC and a parallel line DE which is parallel to side AB and passes through C.

Reseracher: What do you think is the definition of opposite angles?

A1 : angles that are different in two parallel lines and in opposite directions.

Researcher: How about definition of alternate angles?

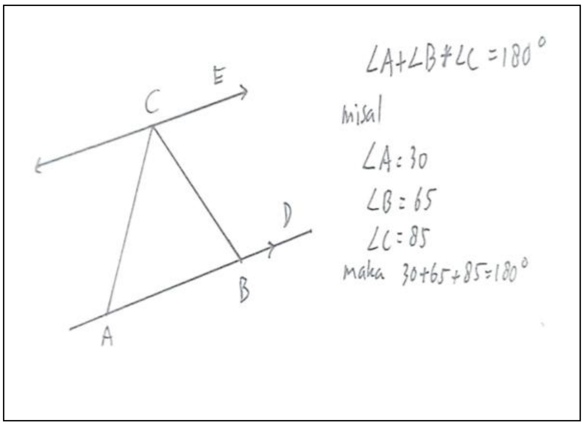
A1 : angles obtained on opposite sides of a transversal whose angles are the same

Researcher: So, your answer is wrong about defition opposite angles?

A1 : yes, it is definition of alternate angles.

In the interview with A1, the student initially believed that the definition of opposite angles was correct, but upon inspection, the student realized that the definition given was for alternating angles.

D2. Students have little intuitive understanding of the concept



Translation:

$\angle A + \angle B + \angle C = 180^0$

Suppose

$\angle A = 30$

$\angle B = 65$

$\angle C = 85$

So that $30 + 65 + 85 = 180^0$

Figure 3. Students have little intuitive understanding of the concept

Based on Figure 3, it can be seen that students use parallel lines but do not know what their uses are (seen in the interview) and students make mistakes in using statements that should be proven but instead make examples in proving them. During the interview, students were asked about their assumptions:

Researcher: is your answer correct?

A2 : Yes, it is correct

Researcher: Can you explain your answer ideas?

A2 : The idea from middle school is that a straight line has a total angle of 180.

Reseracher: Why do you draw with parallel lines? Does it have anything to do with the question?

A2 : I don't know, suddenly the idea emerged spontaneously to make parallel lines.

Researcher: Do you know that the example of the angle you wrote is the one that must be proven?

A2 : No, I don't know.

As seen in the interview above, A2 could not do the proof because he was wrong in writing the statement that should be proven. They use statements by using examples so they cannot prove formally with functional language and use definitions in proving. Thus, it shows that students have little intuitive understanding of the concept.

D3 Student concept image are not sufficient for proof.

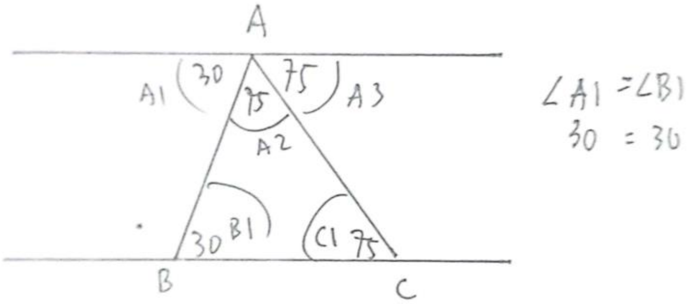


Figure 4. Student concept image are not sufficient for proof

The most common difficulty experienced by students in proving this theorem is that the student's concept of images is still lacking in the process of proving questions (9 students). Based on Figure 4, it can be seen that students only use the language of number symbols and use opposite angles on the answer sheet. However, the student's answer already uses the concept of parallelism which is seen in the student's answer, namely angle A1 = angle B1, which was only revealed in the following interview. This student's answer is already leading to the correct answer, because students can already use the concept of parallelism of two lines and opposite interior angles (seen in the interview). However, from this answer there is a step that has not been fulfilled in the process of completing the proof, namely the linear pair postulate. Students do not complete this step so they cannot conclude the proof process. During the interview, the student was probed about his assumptions:

Researcher: What does angle A1 = angle B1 mean in your answer above?

A3 : because angle A1 and angle B1 are interior alternate angles and because triangle ABC is enclosed by two parallel lines so the angle values are the same.

Researcher: Do you use the concept of two parallel lines and interior alternate angles?

A3 : yes, I used this concept.

Researcher: Why don't you explain the definition of parallel lines and interior alternate angles?

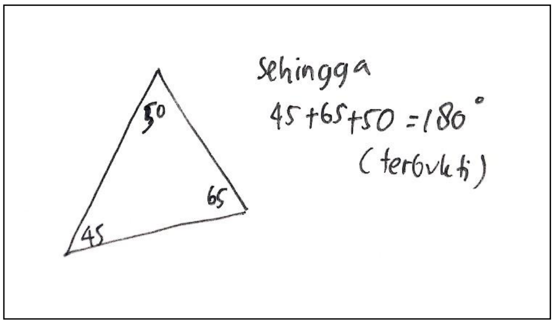
A3 : because it's too long to explain

Researcher: Is it because you can't explain it in language?

A3 : yes

As seen in the interview above, A3 understands the concept in his proof but there are statements related to parallelism and interior angles that are not written in the answer. So A3 has explained the steps in compiling the proof that leads to the correct answer. But after that the students were confused, in concluding the proof process. This is because there is a step that has not been done, namely using the concept of linear pair postulates so that students cannot conclude the answer. This shows that the student's concept of images is still lacking in the process of proving questions.

D4 Students cannot or do not want to build and use their own examples.



Translation:

So that

45 + 65 + 50 = 180^0

(Proved)

Figure 5. Student cannot or do not want to build and use their own examples

Based on Figure 5, it shows that students use inductive proof using the example of the angles in a triangle. However, the correct proof in mathematics is deductive proof so the answer is still wrong. Apart from that, the angle example used is not quite right because the sum of the three angles in the triangle is not equal to 180^0. During the interview, the student was probed about his assumptions:

Researcher: is your answer correct?

A4 : No, it is not correct.

Researcher: Is the sum of the three angles exactly 180?

A4 : Not yet because the third number is 160, not 180.

Researcher: After you have given this example, what steps do you take to ensure that your proof answer is correct?

A4 : By identifying the missing angle, as in the angle is minus 20 to make the total 180. Then add the missing angle to one of the known angles. After adding, make sure the sum of the angles in the triangle is 180.

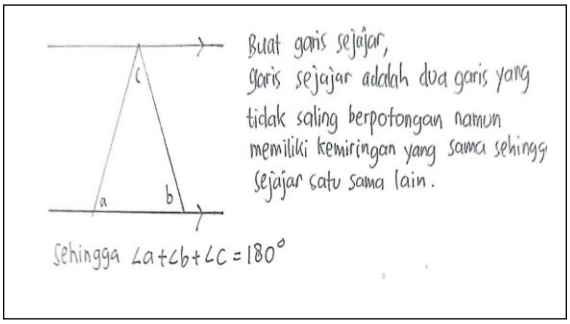
Reseracher: Can you generalize this example?

A4 : No, I cannot.

As seen in the interview above, A4 cannot generalize the examples created so he has difficulty in compiling evidence inductively. When in fact it is necessary to generate and use these examples to understand concepts, definitions, theorems, problems, and notation, and to find proofs. In this case, A4 has not been able to prove it with deductive or formal proof.

D5 Students do not know how to use definitions to determine the overall structure of proof.

Based on Figure 6, it shows that students are correct in showing the definition of parallel lines, namely two lines that do not intersect each other but have the same slope so they are parallel to each other, but they have difficulty continuing the proof after stating this definition. So they can't prove it right.



Translation:

Create parallel lines, parallel lines are two lines that do not intersect each other but have the same slope so they are parallel to each other.

So that ∠a + ∠b + ∠c = 180^0

Figure 6. Students do not know how to use definitions to determine the overall structure of proof

During the interview, the student was probed about his assumptions:

Researcher: is your answer correct?

A4 : No, it is not correct.

Researcher: Is that definition correct?

A4 : yes, Sir

Researcher: What is the next step after writing the definition of parallelism?

A4 : I don't know, Sir
Researcher: What are the two parallel lines used for?
A4 : Maybe, it showed straight corner.

As seen in the interview above, A5 is still unsure about continuing with the next step, even though he has shown the definition of two parallel lines correctly.

D6 Students are unable to understand and use language and mathematical notation.

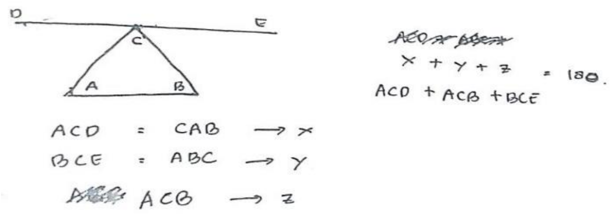


Figure 7. Students are unable to understand and use language and mathematical notation

Based on Figure 7, it shows that students have difficulty making the notation ACD + ACB + BCE symbol, even though they should use the symbol or notation ∠ ACD +∠ ACB + ∠ BCE = 180^0. During the interview, the student was probed about his assumptions:
Researcher: try to explain your answer ACD = CAB? Does that mean angle?
A6 : yes, Sir.
Researcher: Why are the two angles the same?
A6 : Due to the alternate nature of the line AC which cuts parallel lines AB and DE.
Researcher: Does your answer have anything to do with alternate angles?
A6 : Yes, by using the properties of opposite angles I can show that ACD = CAB, and BCE = CDA so that a linear relationship can be formed between the exterior and interior angles and it can be proven that the sum of the angles in a triangle is 180.

As seen in the interview above, A6 already understands the steps in proving, but still has difficulty writing angle symbols. Although after the interview he explained that it was a corner symbol.

D7 Students do not know how to start a proof.

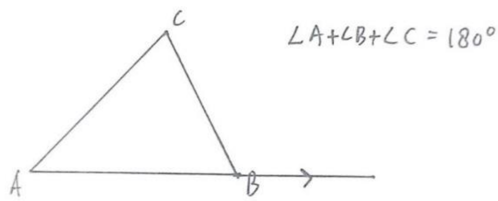


Figure 8. Students do not know how to start a proof

Based on Figure 8, it shows that students have difficulty starting a proof, because they only write questions and draw triangles. During the interview, the student was probed about his assumptions:
Researcher: Why do you rewrite your answer to the question? Don't you understand the meaning of the theorem?
A7 : I was confused, and I didn't continue.
Researcher: Does that mean there are no ideas at all?
A7 : yes, Sir.

As seen in the interview above, A7 had difficulty starting the proof, this could be seen as he answered that he had no idea at all and didn't understand.

Of the seven difficulties experienced by students in proving geometry problems, the biggest difficulty was that students' concept images were inadequate for doing the proofs (9 people). Many students understood the concept to prove the problem, but there were some incomplete statements in compiling the proof. In addition, the statement was not written in the proof even though some students knew and understood the statement. For the smallest difficulty, students were unable to understand and use mathematical language and notation, namely 2 people. This is different from the research by Aziz et al (2024) that students had more difficulty in using definitions to compile a proof in algebra, but in this study, especially proofs in geometry, there were more students with concept images were inadequate for doing the proofs. In geometric proof analysis, students often have more difficulty with the concept of images compared to using formal definitions. Concept image refers to the mental representation that students have of a concept, including visual images, examples, and informal understanding. In contrast, formal definition is a structured and specific description of a geometric concept that can be directly used in the proof process. The problems that students have in using the concept of images are needed in several factors related to understanding, visualizing, and applying geometric concepts abstractly.

One of the main reasons why students have more difficulty with the concept of images is because their visualization and mental interpretation of geometric concepts are often inaccurate or incomplete. This can be seen from Answer A3 showing that students cannot write down the steps of the parallel concept but when interviewed they can explain it. Conformity with the concept image also arises due to students' lack of skills in transforming their visualization or intuition into structured logical arguments (Nurwahyu & Tinungki, 2020). In geometric proofs, images are often used to support understanding, but students must be able to connect the images to the steps of the formal proof. However, if the conceptual picture they have is inadequate or does not match the context of the problem, they will have difficulty constructing valid proofs. Students may understand geometric concepts intuitively through pictures, but difficulties arise when they have to express that understanding with precise mathematical language or connect it with clear deductive steps. This is what makes many students have difficulty in proving this theorem. Then, the findings by Stylianides et al (2024) highlight a significant trend among students in their approach to mathematical proofs. Their research indicates that many students still rely heavily on inductive reasoning when constructing proofs, particularly in the context of numerical examples. This reliance suggests a limited ability to formulate deductive proofs, which is essential for a deeper understanding of mathematical concepts.

The novelty of this research is introducing a new theoretical analysis related to the difficulty in proving the fundamental theorem of geometry, namely Moore's theory. Although previous research related to Moore's theory has been used in algebra, it has not been used in geometry. So the researcher tried to use this Moore's theory analysis in analyzing the difficulty of proof in proving the fundamental theorem of geometry. The limitation of this study is that it does not use technology in proving because technology such as GeoGebra or other visual aids can help students visualize geometric elements, which will improve their understanding of the proof. So using technology is very important in learning mathematics (Cadiz et al, 2024; Suryani et al, 2024). Then preservice teachers should be given a solution through scaffolding to help preservice teachers understand the concept of proof so that students can compile proofs correctly.

CONCLUSION

In conclusion, students' ability to construct mathematical proofs is low, as evidenced by 22 students answering the questions correctly and 36 students answering incorrectly. The most common difficulty involved inadequate concept images (9 students), while the least common difficulty was related to understanding and using mathematical language and notation (2 students). These findings suggest that a more structured and supportive approach is needed to enhance students' proof construction skills and contribute to the understanding of mathematical proof difficulties by highlighting the role of concept images in students' reasoning processes. Based on these findings, a conceptual framework can be developed to guide instructional strategies aimed at strengthening students' conceptual understanding and proof skills. The implications of this study emphasize the need for targeted instructional strategies, such as the integration of technology and scaffolding techniques, to support preservice teachers in mastering mathematical proof construction, particularly in geometry. Future research can explore the use of Geogebra technology as a tool for visualizing and constructing mathematical proofs more effectively, while scaffolding strategies should be further investigated to provide step-by-step guidance, enabling students to develop a deeper understanding of geometric theorems and proof techniques.

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AUTHOR CONTRIBUTIONS

Author 1 creates articles and creates instruments, helps input research data, and is responsible for research, author 2 Analyzes research data that has been collected, author 3 assists in research data analysis and instrument validation.

CONFLICTS OF INTEREST

The author(s) declare no conflict of interest.

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