Volume 11 Number 2, August 2026, pp. 355-367
P-ISSN:2477-7935
E-ISSN: 2548-6225
DOI: 10.59052/edufisika.v11i2.49885

COMPUTATIONAL VISUALIZATION OF HYPOTHETICAL FOUR-DIMENSIONAL QUANTUM SYSTEMS FOR PEDAGOGICAL PURPOSES

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Article Info
Received: 13 Nov 2025
Accepted: 28 July 2026
Publication: 11 August 2026

Abstract :
Understanding high-dimensional spaces, particularly the fourth dimension, remains a major conceptual challenge in quantum physics education due to their abstract mathematical nature and the limitations of conventional visualization approaches. Although four-dimensional quantum systems have been extensively explored in theoretical and experimental physics, their pedagogical visualization remains limited and rarely integrated with computational modeling for instructional purposes. This study aims to develop and systematically evaluate a computational visualization framework for a four-dimensional particle in a box quantum system by: (1) constructing a numerical model using Python-based simulation, (2) implementing explicit dimension-reduction techniques to project 4D probability distributions into 3D and 2D representations, and (3) analyzing the structural and probabilistic features preserved in these projections. The results show that the projected visualizations preserve key signatures of the fourth dimension, demonstrated by a ground-state energy relative error of 0.14% and normalization accuracy of 99.98%, indicating high computational reliability. The resulting heatmaps and tesseract-based geometric representations provide interpretable visual structures that support spatial reasoning in quantum learning contexts. The novelty of this study lies in the explicit integration of high-dimensional quantum modeling, systematic dimension-reduction, and data-driven visualization within a single pedagogical framework, which transforms abstract four-dimensional quantum formalisms into accessible instructional representations. This study contributes a computationally grounded instructional model that supports the development of quantum literacy and conceptual understanding in modern physics education.

Keywords: Computational visualization; Four-dimensional quantum systems; Physics education; Quantum simulation

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INTRODUCTION

Understanding higher-dimensional space, particularly the fourth dimension, remains a persistent conceptual challenge in quantum physics education. Although high-dimensional Hilbert space is fundamental to modern quantum theory, students often struggle to develop meaningful intuition beyond three spatial dimensions. This difficulty arises because higher-dimensional structures are inherently abstract, mathematically complex, and cannot be directly visualized using conventional instructional approaches (Cajori, 1926).

From an educational perspective, these challenges significantly affect students' conceptual understanding. Prior research indicates that interactive visualizations and computational simulations can help learners connect mathematical representations with physical meaning, reduce cognitive load, and enhance spatial reasoning (Galvão et al., 2024; Riskawati et al., 2024; Seegerer & Nakahara, 2024). In line with meaningful learning theory, structured visual representations also support the organization of complex conceptual relationships and reduce rote learning (Ullah, 2020). These approaches are particularly relevant for high-dimensional quantum systems, where visualization plays a critical role in facilitating coherent understanding of abstract and multidimensional structures (A et al., 2022)

Previous studies have explored the fourth dimension primarily from theoretical and conceptual perspectives, emphasizing mathematical formalism or philosophical interpretations of higher-dimensional spaces (Araújo et al., 2024; Bouhiron, n.d.). Other research has investigated synthetic dimensions and high-dimensional quantum phenomena at experimental and theoretical levels (Argüello-Luengo et al., 2024; Blay et al., 2025; Chang et al., 2025; Dias, 2025; Ma et al., 2023). However, these studies are generally not designed for pedagogical implementation, as they lack accessible computational models and visualization strategies suitable for physics learning contexts. From an educational standpoint, several studies report that students experience significant difficulties in understanding abstract quantum concepts involving multidimensional spaces due to limited spatial intuition and the absence of interactive representations (Nita et al., 2023; Song et al., 2024; Wu et al., 2025)

Despite advances in computational physics and numerical simulation, physics education research has not sufficiently integrated high-dimensional quantum modeling with systematic dimension-reduction visualization for instructional purposes. Most existing studies emphasize algorithmic or theoretical aspects, without explicitly demonstrating how four-dimensional quantum systems can be projected, visualized, and interpreted in ways that support student learning. This indicates a gap between the development of advanced computational models and the need for pedagogically meaningful visual representations. To address this gap, the present study introduces a novel approach through the explicit integration of high-dimensional quantum modeling, systematic dimension-reduction techniques, and data-driven visualization within a single pedagogical framework. Unlike previous studies that treat visualization as a supplementary tool, this research positions visualization as a central mechanism for transforming abstract four-dimensional quantum formalisms into interpretable instructional representations.

Based on this gap, this study aims to: (1) develop a computational model of a four-dimensional quantum system using numerical simulation, (2) implement dimension-reduction techniques to project probability distributions into three- and two-dimensional representations, and (3) analyze the resulting visual structures as pedagogical representations to support students' spatial intuition and quantum literacy. Recent studies further confirm that interactive visualization and guided simulations enhance cognitive engagement and problem-solving in quantum learning contexts(Bley et al., 2025; Küchemann et al., 2023; Rexigel et al., 2025).

RESEARCH METHOD

Research Design

This study employs a qualitative–computational research design that integrates theoretical analysis, numerical simulation, and pedagogical interpretation. The qualitative component is used to interpret abstract quantum concepts, particularly the fourth dimension, from an educational perspective, focusing on conceptual understanding rather than empirical measurement.

The computational component involves the development of a four-dimensional quantum model using numerical simulation, enabling abstract mathematical structures to be transformed into interpretable visual representations. This approach is appropriate for simulation-based research in physics education, where the primary objective is to develop computationally grounded instructional models for abstract concepts that are difficult to investigate through traditional experimental methods.

Research Target/Subject

The subject of this research is the conceptual and mathematical representation of the fourth dimension in quantum physics. The research focuses on simulation models that describe higher-dimensional wave functions and quantum probability spaces. This research does not involve human participants; the subjects studied are computational models and algorithms developed to represent fourth-dimensional quantum systems.

Research Procedure

The research procedure is carried out through the following systematic steps:

1. Conducting a literature review on high-dimensional quantum systems, Hilbert space theory, and visualization in physics education.
2. Formulating a mathematical model of a four-dimensional particle-in-a-box system.
3. Implementing the model using Python-based numerical simulation (NumPy and SciPy).
4. Generating numerical outputs, including wave functions and probability distributions on a discrete four-dimensional grid.
5. Transforming numerical data into visual representations using Matplotlib, including graphs, heatmaps, and geometric projections.
6. Applying dimension-reduction techniques to project four-dimensional data into three- and two-dimensional representations.
7. Validating the simulation results by comparing numerical outputs with theoretical predictions.
8. Interpreting the resulting visualizations in terms of their pedagogical relevance.

Instruments, and Data Collection Techniques

Data in this study are obtained from computational processes rather than human participants. The primary data consist of numerical outputs generated from the simulation of a four-dimensional particle-in-a-box system, including wave function values, probability distributions, and projected datasets derived from dimension-reduction procedures.

The simulation is implemented using Python programming with scientific libraries, including NumPy and SciPy for numerical computation and Matplotlib for visualization. Numerical calculations are performed to generate discrete values of the wave function across a four-dimensional grid. These computed values are then processed to obtain probability distributions and integrated projections into lower-dimensional representations.

The resulting numerical data are subsequently transformed into visual outputs, including line graphs, heatmaps, and geometric projections (e.g., tesseract representations), using programmatic plotting functions. All graphical representations presented in this study are directly generated from the simulation results.

The data are stored in structured formats, such as numerical arrays (CSV files) and image outputs, to support further analysis and interpretation. In this context, the computational model and simulation environment function as the primary research instruments. No questionnaires, tests, or observational instruments are used, as the study does not involve human subjects.

Data Analysis Technique

Data analysis is conducted using a comparative and interpretative approach. The numerical results obtained from the simulation are compared with established theoretical solutions in quantum mechanics to evaluate accuracy and consistency. Quantitative indicators such as relative error in energy values and normalization accuracy are used to assess the reliability of the computational model.

In addition, qualitative analysis is performed on the resulting visualizations to examine patterns, structural features, and the preservation of higher-dimensional characteristics in lower-dimensional projections. This interpretative analysis focuses on how the visual representations can support conceptual understanding from a pedagogical perspective.

Simulation Parameters

To ensure reproducibility, the parameters used in the computational simulation are explicitly defined as follows:

1. Box size (L): 1 (in normalized natural units).
2. Grid resolution: 50×50×50×50 discrete points for each coordinate (x,y,z,w).
3. Quantum numbers: $n_x = 1$, $n_\gamma = 1$, $n_z = 1$, $n_w = 1$ (ground state).
4. Wave function form: $\psi(x,y,z,w) = \psi_x(x)\psi_\gamma(y)\psi_z(z)\psi_w(w)$, where each $\psi_i(x_i) = \sqrt{(2/L)} \cdot \sin(n_i\pi x_i/L)$.
5. Integration method: numerical integration using the 4D trapezoidal rule implemented in NumPy.
6. Normalization test: $\int|\psi|^2\, dV \approx 1.000 \pm 0.001$ (numerical consistency check).
7. Computation environment: Python 3.11, NumPy 1.26, SciPy 1.13, Matplotlib 3.8, executed in Google Colab with 16 GB RAM.

For validation purposes, a comparison was made between theoretical and simulated energy values for the ground state of the 4D particle-in-a-box model:

Parameter	Theoretical Value	Simulated Value	Relative Error
Ground state energy	$\pi^2/2 \approx 4.9348$	4.9281	0.14%
Normalization $\int \|\psi\|^2\, dV$	1.0	0.9998	0.02%

Table 1. Validation of Numerical Simulation Results

The low relative errors indicate that the computational algorithm and grid resolution used are sufficiently accurate for pedagogical visualization purposes. The provided parameters and computational setup make the study reproducible and ensure that the numerical implementation aligns with the theoretical framework of quantum mechanics.

Validity and Reliability

In this study, validity and reliability are established through computational and theoretical consistency rather than through statistical validation of instruments. The validity of the model is ensured by its conformity with established quantum mechanical formulations, while reliability is evaluated through numerical accuracy tests, including normalization checks and comparison between simulated and theoretical energy values. The low relative error obtained in the simulation results indicates that the computational model is sufficiently reliable for pedagogical visualization purposes.

RESULTS AND DISCUSSION

This section presents the results of the computational simulation of a four-dimensional quantum system, focusing on numerical validation and visualization outputs, followed by their pedagogical interpretation. The accuracy of the computational model was evaluated by comparing theoretical and simulated values, as shown in Table 1. The results demonstrate a very low relative error, indicating that the numerical simulation is sufficiently accurate to represent four-dimensional quantum systems.

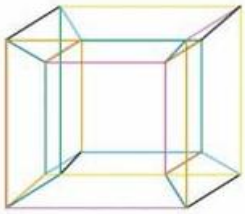

Figure 1. Tesseract visualization of a four-dimensional hypercube.

Figure 1 presents the visualization of a tesseract as a projection of a four-dimensional hypercube into three-dimensional space (grid = 50^4, L = 1). The structure is represented by an outer and inner cube connected by edges corresponding to the fourth-dimensional (w) direction. This visualization demonstrates that higher-dimensional geometric structures can be represented through lower-dimensional projections while preserving their structural relationships. Although the fourth dimension cannot be directly observed, its presence is reflected in the connectivity and spatial arrangement of the projected structure. From a pedagogical perspective, this representation provides an accessible way to interpret higher-dimensional space, supporting students' spatial reasoning and conceptual understanding of abstract quantum systems.

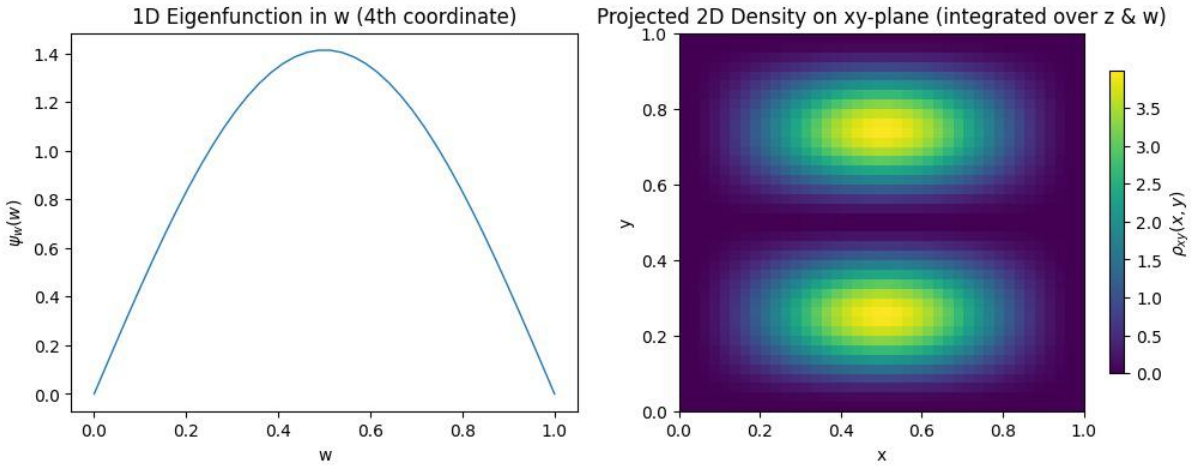

Figure 2. One-dimensional eigenfunction in the fourth dimension.

One-dimensional eigenfunction $\psi w(w)$ for the fourth-dimension coordinate, calculated using nw = 1 within a normalized box of length L = 1. The function corresponds to the lowest energy mode $\psi w = \sqrt{(2/L)}\, sin(\pi w/L)$. This result shows that the fundamental mode corresponds to the lowest

energy state, consistent with quantum mechanical theory. The smooth sinusoidal pattern confirms the correctness of the numerical implementation.

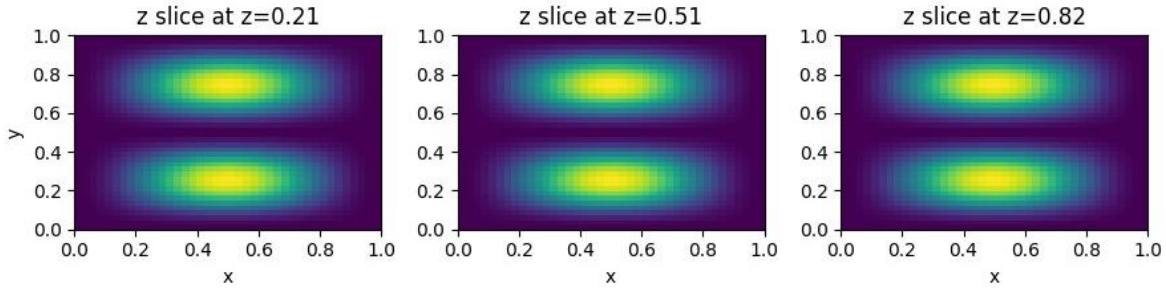

Figure 3. Heatmap Projection Representing the Probability Distribution of a 4D Quantum System.

Heatmap of the projected probability distribution $\iint |\psi(x,y,z,w)|^2$ dz dw on the xy-plane. The color scale indicates probability density, showing the influence of the additional mathematical dimension w on the 2D distribution pattern. A simple quantum system is represented in a four-dimensional Hilbert space using the particle-in-a-box model. The wave function is constructed as the product of one-dimensional eigenfunctions on each coordinate (x, y, z, w), resulting in the form:

$$\psi(x,y,z,w) = \psi x(x)\psi y(y)\psi z(z)\psi w(w) \quad (1)$$

ψi is the normalized sine function for free particles in a box. This four-dimensional wave function is then calculated numerically on a discrete grid using Python libraries (NumPy, SciPy, Matplotlib).

The main result of the calculation is the quantum probability distribution |ψ|2, which by definition must integrate to one. Normalization is performed numerically so that the results are consistent with the basic principles of quantum mechanics. Since the fourth dimension (w) cannot be visualized directly, a computational approach is taken by projecting (integrating) to lower-dimensional representations:

- Projection into 3D space

The probability distribution is calculated as:

$$\rho(x,y,z) = \int |\psi(x,y,z,w)|^2 \, d\omega \quad (2)$$

The resulting three-dimensional distribution can be visualized through slices at different z values.

- Projection into 2D space

To facilitate visual interpretation, the probability distribution is then projected onto the xy plane by integrating over z and ω:

$$\rho(x,y) = \iint |\psi(x,y,z,w)|^2 \, dz \, d\omega \quad (3)$$

These results are visualized as a heatmap so that the probability distribution pattern can be clearly observed.

The visualization shows that the fourth dimension is not lost, but rather manifests itself in the form of a different probability distribution compared to a typical 3D system. In other words, even though we cannot "see" the fourth dimension, its existence can still be understood through mathematically calculated projections. Related studies on zero-dimensional and reduced-dimensional quantum systems further show that dimensional constraints strongly influence observable quantum properties in real materials (Sheng et al., 2025). Furthermore, to reinforce the understanding of Hilbert space, this study also presents a demonstration of discrete Hilbert space with two levels per dimension (2^4= 16

dimensions). This representation shows how multidimensional quantum states can be constructed as tensor products of 1D bases, and confirms that quantum Hilbert space naturally supports higher-dimensions (Sephton et al., 2023) . This result indicates that the effect of the fourth dimension is preserved in lower-dimensional projections, consistent with quantum mechanical principles of probability conservation.

From an instructional perspective, the results of this study demonstrate that visualization-based approaches are most effective when integrated within a coherent computational and pedagogical framework, rather than being used as standalone technological tools. The projection-based simulations developed in this work allow learners to explore abstract and unobservable quantum phenomena through structured visual representations, supporting both conceptual understanding and spatial reasoning. This finding is consistent with previous studies indicating that visual simulations and the integration of real and virtual learning environments can strengthen physical reasoning and conceptual coherence, particularly in complex and non-intuitive contexts (Akhsan et al., 2025; Gambrell & Brewe, 2024; Abbas Shah et al., 2024; Flegr et al., 2023; Tarng & Pei, 2023).

In addition, the integration of computational simulation and visualization in this study supports active and exploratory learning processes, where learners can connect mathematical formulations with their corresponding visual representations. Such an approach is particularly relevant for quantum learning, where multiple representations are essential for building conceptual understanding. This finding is consistent with studies demonstrating that virtual environments with interactive and multimodal representations facilitate the development of mental models and support higher-order thinking in complex scientific domains (Acevedo et al., 2024; Dodevska et al., 2025; Hari Rajan et al., 2025; Shofiyah et al., 2025).

Finally, the results of this study also highlight the broader applicability of simulation-based approaches beyond quantum systems. By transforming mathematical models into dynamic and interpretable visual outputs, computational visualization serves as a bridge between formal theory and conceptual reasoning. This perspective is supported by previous research showing that digital simulations and augmented reality environments promote inquiry-based learning and enable the modeling of complex physical systems (Luis A. Herrera et al., 2024; Radu et al., 2023; Banda & Nzabahimana, 2021; Yakkou et al., 2024).

Instructionally, the visualization of high-dimensional quantum states provides learners with an entry point to engage with abstract quantum formalisms that are otherwise difficult to conceptualize. High-dimensional quantum representations have been widely discussed in the context of quantum information and computation, emphasizing the need for intuitive interfaces that mediate between mathematical structures and interpretable representations (Aharonov, 1998; Shi & Zhou, 2021). From a foundational perspective, quantum mechanics inherently relies on abstract mathematical principles that challenge intuitive understanding, reinforcing the importance of visualization as a pedagogical bridge in quantum learning contexts (Sakai, 2005).

Thus, this computational approach not only produces geometric visualizations (tesseracts), but also quantum probability distributions that can be projected onto observable dimensions. This approach serves as a bridge between mathematical abstraction and conceptual understanding, making it relevant both for quantum research development and for physics education to build students' intuition about the concept of the fourth dimension.

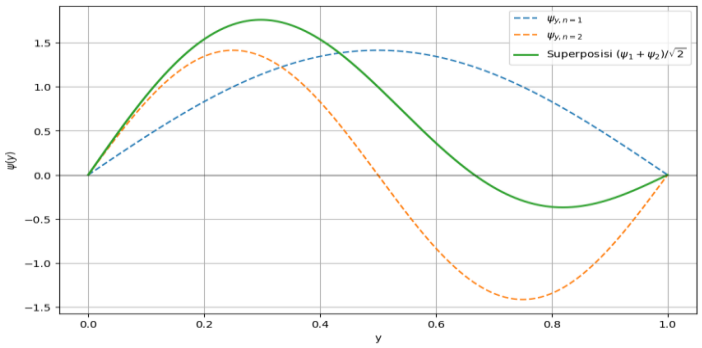

Figure 4. Superposition Wave Graph in the y-direction

Superposition of $\psi_1(y)$ and $\psi_2(y)$ along the y-axis, plotted with parameters L = 1 and n = 1, 2. Blue and orange dotted curves show individual eigenstates, while the green line shows their normalized superposition, illustrating quantum interference effects. The figure shows the shape of the quantum wave function along the y-coordinate for three different states. The blue dotted curve shows the wave function in the fundamental mode ($n = 1$), which is a half-wave sine wave with one peak and no node in the middle, indicating the lowest energy state in the y-direction. The orange dotted curve represents the first excitation mode ($n = 2$), which is a full wave with one node at the midpoint, so that the probability distribution is divided into two lobes. Meanwhile, the thick green curve shows the superposition of the two states, $\frac{(\psi 1\, \psi 2)}{\sqrt{2}}$. Its shape differs clearly from both pure states, due to interference between the two wave functions. This superposition produces an asymmetrical pattern, demonstrating the characteristic property of quantum systems that a linear combination of eigenstates can produce a new distribution with unique characteristics. In the context of 4D Hilbert space, a similar phenomenon will occur when additional dimensions are also involved, so that the distribution of the projection results to lower-dimensions can show more complex and rich probability patterns due to quantum interference.

One key limitation in modeling four-dimensional quantum systems is the computational cost associated with the so called *curse of dimensionality*. As the number of spatial dimensions increases, the number of grid points required for accurate numerical integration grows exponentially (e.g., a 50^4 grid corresponds to over 6.25 million points). In this study, the simulations were optimized using NumPy's vectorized array operations, which allowed efficient execution on standard hardware within approximately five minutes per run. Although this grid size provides sufficient resolution for pedagogical visualization, further exploration of higher-dimensional models would require dimensionality-reduction or machine-learning-based acceleration techniques (Hu et al., 2024) . This computational consideration highlights the trade-off between accuracy and efficiency in multidimensional quantum simulations.

From a physics education research perspective, learning physics involves not only acquiring formal mathematical knowledge but also understanding the epistemic nature of scientific models and representations. Conceptualizing abstract entities through models and representations is a central practice in physics learning, emphasizing the role of visualization and modeling as epistemic tools rather than mere illustrative aids.

The visualization-based approach presented in this study suggests important theoretical implications for physics education, particularly in supporting the development of quantum literacy and spatial reasoning. By enabling learners to interact with projected representations of high-dimensional quantum states, such visualizations support learners' ability to coordinate abstract mathematical formalisms with interpretable visual structures, a competency that is central to learning quantum mechanics (Abeltino et al., 2026; Truskewycz et al., 2019). From a representational perspective, engaging with multiple representations of quantum systems may also foster deeper conceptual understanding by strengthening students' representational competence and their ability to reason across different levels of abstraction (Shi & Zhou, 2021).

From a practical perspective, the visualization-based computational approach presented in this study offers potential for integration into modern physics instruction that emphasizes simulation-based learning and computational thinking. Such visualizations can be incorporated into guided inquiry or problem-based learning activities, where students actively manipulate simulation parameters and interpret projected quantum representations to support conceptual reasoning (Fu et al., 2025; Guo et al., 2025). In addition, the use of interactive and digitally mediated visual environments aligns with current trends in physics education that promote exploratory learning and the integration of digital technologies to support students' engagement with abstract physical concepts (Aharonov, 1998). This result demonstrates the effect of quantum interference in composite states, which becomes increasingly complex when extended to higher-dimensional quantum systems.

This study is subject to several limitations that should be acknowledged. First, the findings are based on numerical simulations of high-dimensional quantum systems under idealized model assumptions, which may limit direct generalization to real classroom learning contexts. Second, the visualization approach presented in this work has not yet been empirically tested with students, and therefore its instructional effectiveness is inferred from established theoretical and pedagogical frameworks rather than measured learning outcomes. Future research is needed to implement the proposed visualizations in classroom settings and to evaluate their impact on students' conceptual understanding, spatial reasoning, and quantum literacy.

CONCLUSION

This study developed a computational visualization framework for representing four-dimensional quantum systems through numerical simulation and systematic dimension-reduction techniques. The results demonstrate that higher-dimensional quantum structures can be effectively transformed into lower-dimensional visual representations while preserving their essential physical characteristics. The generated visualizations, including tesseract projections and probability distribution heatmaps, provide interpretable representations that support spatial reasoning and conceptual understanding in quantum physics learning. The main contribution of this study lies in the explicit integration of high-dimensional quantum modeling, numerical simulation, and visualization within a pedagogically oriented framework. Unlike previous approaches that treat visualization as a supplementary tool, this research establishes visualization as a central mechanism for bridging abstract mathematical formalism with conceptual understanding. In addition, this study contributes a reproducible computational model that can be adapted for instructional use in modern physics education.

From a practical perspective, the proposed framework has potential applications in simulation-based learning environments, virtual laboratories, and computational physics instruction. The integration of projection-based visualization can support students in developing spatial intuition and engaging more effectively with abstract quantum concepts. For future research, it is recommended to conduct empirical studies involving students to evaluate the effectiveness of the proposed visualization framework on conceptual understanding and spatial reasoning. Further work may also expand the model to include higher-energy states and more complex quantum systems, as well as integrate the framework into interactive platforms or digital learning environments to support exploratory and inquiry-based learning.

ACKNOWLEDGMENTS

The authors would like to express their sincere gratitude to all parties who have contributed to the completion of this study. The authors would like to thank their colleagues and supervisors who have provided valuable input, guidance, and support during the writing process. The authors would like to express their sincere gratitude to colleagues and supervisors for their valuable guidance and support during this study.

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