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APPLICATION OF BESSEL EQUATION TO CIRCULAR SLIT FRAUNHOFER DIFFRACTION: A SYSTEMATIC REVIEW

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Abstract :
This article aims to introduce the development of a simulation model using matlab in the calculation of the bessel function to describe the fraunhofer diffraction pattern in a circular slit. This research method uses the Systematic Literature Review (SLR) method equipped with PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) guidelines, this method involves searching, filtering, and analyzing literature from reliable sources to compile a comprehensive theoretical framework. Based on the simulation of circular slit diffraction using matlab application with the same wavelength and different radii of the circle, it shows that the larger the radius of the circle will produce a narrower diffraction pattern, while the smaller the radius of the circle will produce a wider diffraction pattern. Meanwhile, when using the same radius and different wavelengths, it shows that longer wavelengths produce wider diffraction patterns, while shorter wavelengths produce narrower diffraction patterns.

Keywords: Bessel Funciton, Diffraction, Fraunhofer

INTRODUCTION

Partial differential equations (PDPs) are one of the most widely used mathematical tools for modeling complex natural, engineering and social phenomena that change in space and time. However, in reality, solving PDPs is often difficult, especially when approximating complex contours and unforced boundary conditions. It is possible because the Bessel function is the solution of the PDP and it is an important topic in applied mathematics. The function was introduced by German astronomer and mathematician Friedrich Wilhelm Bessel in the 19th century while reviewing phenomena in astronomy, but later this function was used in many fields of science. Optics is a branch of physics that studies the properties of light. One of the difficult things in the study of optical materials is the geometry of optical material (Pradana et al., 2017). In 1897, Durnin was the first to discover Bessel rays (Stoyanov et al., 2020), which had originally been studied by Zel'dovich and McLeod, where it was stated that the zero-order Bessel function was modeling nondiffracting light rays with amplitude profiles (Karahroudi et al., 2017).

In-depth study of Bessel functions (Heitman et al., 2015), can improve the precision of high intensity (Grunwald & Bock, 2020). Bessel functions can be applied to various fields such as optics in the application of Bessel (muller) beams, Fractional Vortex Beams (FVBs) (Peters et al., 2022), propagation of wave properties (Borghi, 2022), and also in Fraunhofer diffraction on diffraction pattern analysis (Porfirev et al., 2021) so that the wave intensity can be described through diffraction patterns (Korolenkoo, 2020). The Bessel equation is divided into two, namely the first type of Bessel function $Jn(x)$ and the second type $Yn(x)$. With the general form of the differential equation $x^2\,y'' + xy' + (x^2 - p^2)y = 0$. And the general solution $y(x) = C_1\,J_n(x) + C_2\,Y_n(x)$

1. Bessel Function of the First Kind
   Bessel Function of the First Kind $J_n\,(x)$ is defined as:
   $$J_n(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)\Gamma(n+p+1)} \left(\frac{x}{2}\right)^{2n+p} \tag{1}$$

2. The Second Type of Bessel Function
   The second type of Bessel function $Y_n(x)$ is defined as:
   $$Y_n = -\frac{\left(\frac{x}{2}z\right)^2}{\pi} \sum_{k=0}^{n-1} \frac{(n-k-1)!}{k!} \left(\frac{1}{4}z^2\right)^k + \frac{2}{\pi} \ln\left(\frac{1}{2}z\right) J_n(z) - \frac{\left(\frac{x}{2}z\right)^n}{\pi} \frac{\left(\frac{1}{4}z^2\right)^k}{k!\,(n+k)!'} \tag{2}$$
   $$\sum_{k=0}^{\infty} [\psi_0\,(k+1) + \psi_0(n+k+1) \quad \frac{\left(\frac{1}{4}z^2\right)^k}{k!(n+k)!'}$$

Bessel rays have the characteristic of maintaining their shape despite scattering, this scattering is usually known as diffraction-free properties. With its characteristics, Bessel rays are indispensable in various applications. For example, imaging, detection and manipulation of light on a microscopic scale (Saadati-Sharafeh et al., 2020). Bessel rays also have a very unique ability, Bessel rays can remain focused when moving far away which is called diffraction-free. This ability of Bessel light is different from ordinary light waves which, when moving away from the source, will experience scattering and broadening (Khonina et al., 2020). For applications, such as microscopy or optical capture (Suzuki, 2020), the Bessel function can be utilized to reduce the interference of an out-of-focus dye into a sharper image. This allows for precise control of small particles (Ni et al., 2017). Usually, the Bessel function is often also used to describe an intensity distribution in the Frounhofer diffraction pattern (Siemion, 2021).

In this article, we discuss the deflection or propagation of waves when they pass through or around a gap or even a specific barrier. This wave propagation can occur when the gap size is equal to the wavelength. These barriers can be single slits or multiple slits, can also be straight or small circular openings and have patterns consisting of dark and light (Anggur et al., 2019). There are two types of diffraction that have long been considered by scientists, namely Fresnel diffraction and Fraunhofer diffraction. Fresnel diffraction occurs when the point source of the wave is located far from the slit; at the same time, the diffraction pattern is observed at close range. In contrast, we will talk about Fraunhofer diffraction when the deflected rays are parallel. The zone close to the region of significant wave inhomogeneity is called the Fresnel zone, while the more distant zone, where the waves become almost homogeneous (quasi-homogeneous) and the ray approximation can be used, is called the Fraunhofer diffraction zone.

Diffraction is considered to be one of the main manifestations of the wave nature of light (Budak et al., 2020). Frouhofer diffraction provides an analytical and in-depth approach used to study intensity distributions involving Bessel functions (Liao et al., 2020). The completion of Fraunhofer diffraction is very difficult as it requires a strong understanding of mathematics and physics, also it can be said about many aspects in this optics course. Diffraction is defined as propagation through a narrow gap (Trisnowati et al., 2022). Fraunhofer diffraction has an infinite range which refers to the phenomenon of light passing through small holes or obstacles and creating predictable patterns in the far field (Fu, 2023). In diffraction theory, there are two approaches, namely near-field and far-field approaches. If the distance is very far, it will use the far-field approach, which will produce a Fraunhofer diffraction pattern. On the other hand, if the distance is not too far away, then use the near field and will produce a Franciscan diffraction pattern.

Diffraction is not experimentally subjected to a bessel beam, causing diffraction to be limited (Chillara et al., 2019). Diffraction of light waves is a phenomenon often seen in everyday life and also in scientific experiments Since the discovery of light wave behavior, diffraction has been a cornerstone in optics. The teaching of diffraction theory is usually done theoretically based on a mathematical approach that can hinder the understanding of physical phenomena (Zapata Valencia et al., 2023). To maintain the Fraunhofer diffraction intensity calculation, we need to start by calculating the amount of interference optics on the screen; this procedure requires the use of Bessel functions.

This topic is complex and generally requires laboratory work to solve each detailed case (Yanuarief, 2016). In many circumstances, physics is difficult to learn without direct guidance. Indeed, many students who are prominent today have faced challenges in the past, such as the problem of understanding the operation of concepts (Winarti, 2021). At its most difficult, the circumstances are worth doing so that a particular experiment is declared comprehensible. For example, optics, diffraction are experimental subjects (Setyono et al., 2016). In this article, we will focus on Fraunhofer diffraction and how it works in simulations.

Previous studies, such as those published by Trisnowati E. et al (2022) and Khachatrian A.Zh. (2021) on Fraunhofer diffraction. Trisnowati E. et al (2022) in their research entitled "Distribution of the Fraunhofer Diffraction Intensity by a Rectangular Slit Using a Razor Blade" focuses on a simple experiment that uses a razor blade to create a square slit to measure the intensity of light in the Fraunhofer diffraction pattern. This research generates data through direct experimentation, but has limitations on accuracy and does not allow for systematic analysis of additional parameters such as wavelength or slit radius. On the other hand, Khachatrian A.Zh. (2021) in his research titled "The Fraunhofer Pattern of a Wave Field Generating by a System of Coherent Emitting Point Sources" focuses on a theoretical approach in analyzing the superposition of waves generated by many coherent sources. However, these two studies have not utilized technology for simulation and visualization of Fraunhofer diffraction.

The utilization of technology such as Matlab applications, has accurate and efficient numerical computing capabilities in the calculation of Bessel functions for large values or complex simulations. in addition, the use of Matlab applications also introduces a more practical alternative in the calculation of Bessel functions in Fraunhofer diffraction. Fraunhofer diffraction simulation using Matlab application looks simple, but this simulation is able to provide ease of learning, time efficiency, increase creativity and cost savings without the need to provide experimental equipment. This research aims to introduce a more practical alternative in calculating the Bessel function in Fraunhofer diffraction, develop a simulation model using Matlab that can calculate and describe diffraction patterns in circular slits and contribute to the development of computing for calculation and simulation in the field of physics. This research aims to introduce a more practical alternative in the calculation of Bessel functions in Fraunhofer diffraction, develop a simulation model using Matlab that can calculate and describe diffraction patterns in circular slits and contribute to the development of computing for calculations and simulations in the field of physics.

RESEARCH METHOD

The method used in writing this article is Systematic Literature Review (SLR) with PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) design.

1. Systematic Literature Review (SLR)

   SLR or Systematic Literature Review is a structured and methodological approach in conducting research with the aim of collecting, identifying, evaluating, and critically synthesising all research studies relevant to a particular topic or research question (Febrianti, 2024). The source search was limited to research in the form of articles. The articles used were research that had been analysed and published in English and Indonesian journals. In managing articles obtained from online databases, the authors used Mendeley Desktop as reference management software and PDF, a programme developed by Elsevier to manage references, share articles, and search research literature online. In the identification stage, literature searches were conducted through two main sources, namely Google Scholar and Taylor and Francis. with a focus on the application of the Bessel equation between 2015-2025.

2. PRISMA

PRISMA is a tool that helps authors to report systematic reviews and meta-analyses clearly and completely. The aim is for authors to present research transparently, so that the benefits of the research can be properly assessed. PRISMA focuses on how to report research in a mo (Sastypratiwi & Dwi Nyoto, 2020). This research follows the PRISMA steps, namely: developing research questions, conducting a literature search, assessing study quality, collecting data, analysing and presenting results, interpreting findings, and updating the review. Screening will be conducted independently by four authors to ensure the validity of the results. Articles that pass the initial selection will be examined based on predetermined inclusion criteria, such as topic suitability, publication type, and data availability. Articles that do not fulfil the criteria will be excluded. The collected data will then be analysed qualitatively to find the main themes, categories, and patterns in the application of the Bessel equation.

Table 1. Inclusion and Exclusion Criteria

Criteria	Description
Inclusion	1. The chosen research subject should be relevant to the application of the Bessel equation. 2. Publications published in the time span between 2015 - 2025. 3. Articles are selected from journals indexed in reputable academic databases. 4. The population under study must be related to the technology or application of the Bessel equation.
Exclusion	1. The chosen research subject is not relevant to the application of the Bessel equation. 2. Publications published outside the timeframe between 2015 - 2025. 3. Selected articles are not from journals indexed in reputable academic databases. 4. The population under study is not related to the technology or application of Bessel equation.

Table 2. review selection results based on criteria

Criteria	Inclusion	exclusion	Reason
Software	Google Scholar and Taylor and Francis.	Other non-indexed sources.	To ensure the journal is trusted and verified.
Accessible	Accessible	Inaccessible	To ensure research is easily accessible.
Year of publication	Articles published within the last 10 years, i.e. 2015-2025.	Articles published under 2015.	Articles with publications within the last 10 years ensure research remains current and relevant.
Language	English and Indonesian.	In addition to English and Indonesian.	English is the international language, while Indonesian is the official language of the author.

Table 3. Bessel function application findings (relevant journals)

Author	Title	Citation	Year	Publisher
Azlan et al	Teaching and learning of postgraduate medical physics using Internet-based e-learning during the COVID-19	345	2020	*Physica Medica*

	pandemic–A case study from Malaysia.			
Karahroudi et al	Generation of perfect optical vortices using a Bessel–Gaussian beam diffracted by curved fork grating	36	2017	*Applied optics*
Khonina et al	Bessel beam: Significance and applications—A progressive review	216	2020	*Micromachines*
Ni, J et al	Three-dimensional chiral microstructures fabricated by structured optical vortices in isotropic material	288	2017	*Light: Science & Applications*
Pradana, S. D. S., Parno, & Handayanto, S. K.	Pengembangan tes kemampuan berpikir kritis pada materi Optik Geometri untuk mahasiswa Fisika.	80	2017	*Jurnal Penelitian dan Evaluasi Pendidikan*
Setyono, A., Nugroho, S. E., & Yulianti, I.	Analisis kesulitan siswa dalam memecahkan masalah fisika berbentuk grafik.	38	2016	*UPEJ Unnes Physics Education Journal*
Saadati-Sharafeh, F., Borhanifar, A., Porfirev, A. P., Amiri, P., Akhlaghi, E. A., Khonina, S. N., & Azizian-Kalandaragh, Y.	The superposition of the Bessel and mirrored Bessel beams and investigation of their self-healing characteristic.	26	2020	*Optik2*
Chillara et al	Ultrasonic Bessel beam generation from radial modes of piezoelectric discs	24	2019	*Ultrasonics*
Wu, Z., Wang, X., Sun, W., Feng, S., Han, P., Ye, J., ... & Zhang, Y.	Vectorial diffraction properties of THz vortex Bessel beams.	39	2018	*Optics Express*
Anggur et al	Kajian Komputasi Numerik Model Integratif pada Difraksi Celah Lingkaran Menggunakan Metode Pendekatan Simpson 1/3	6	2019	Jurnal Fisika: Fisika Sains Dan Aplikasinya

Discussion of table 3

Based on the data from the table above, it describes journal publications from 2015-2025. Database determination, data information in this study are journals and articles obtained through online electronic academic data-based portals such as Taylor and Francis and Google Scholar. Where, the keyword used in the SLR method is Bessel function for circular slit Fraunhofer diffraction resulting in 2,033 journals. The increase in the number of journal publications from 2015-2025 indicates an increase in interest in the field. the following is a rician of the increase.

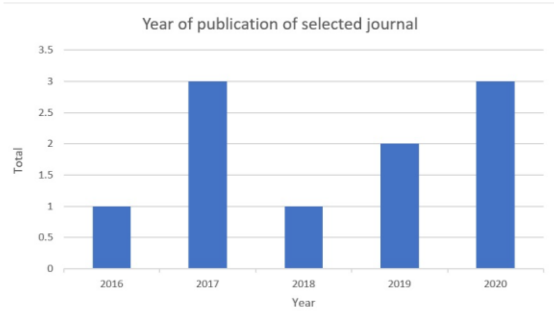


Figure 1. Year of publication of selected journal

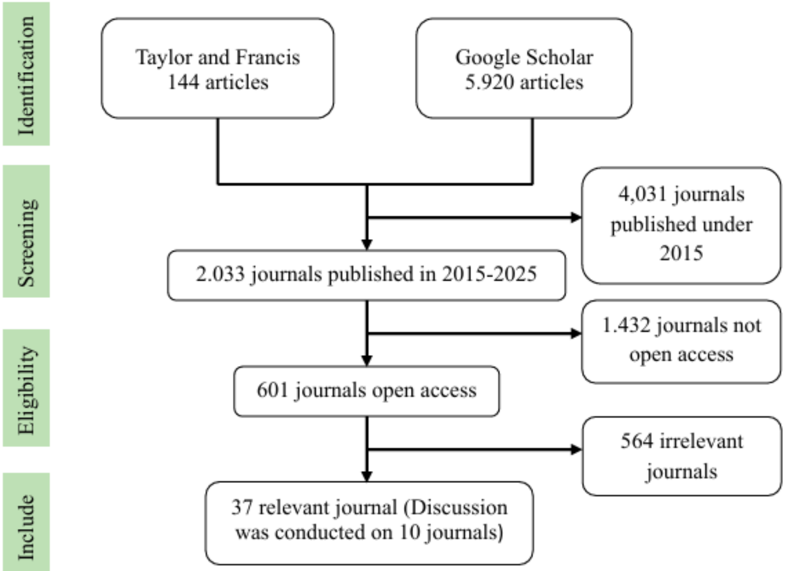


Figure 2. PRISMA stage

RESULTS AND DISCUSSION

The diffraction pattern consists of a bright center surrounded by alternating dark and bright lines. The bright areas occur due to overlapping light waves in the same phase, while the dark areas occur due to overlapping light waves with different phases. Fraunhofer diffraction creates a more structured pattern and is easier to observe compared to Fresnel diffraction because, at large distances, the light reaching the screen is almost parallel. For circular slits, Fraunhofer diffraction plays an important role in the development of optical instruments, such as increasing the resolution of telescopes (Al-Kahfi & Yanuarief, 2019).

$$I = I_0 \left[\frac{2J_1\left(\frac{k.a.q}{R}\right)}{\frac{k.a.q}{R}} \right]^{2n+p} I_0 \qquad (3)$$

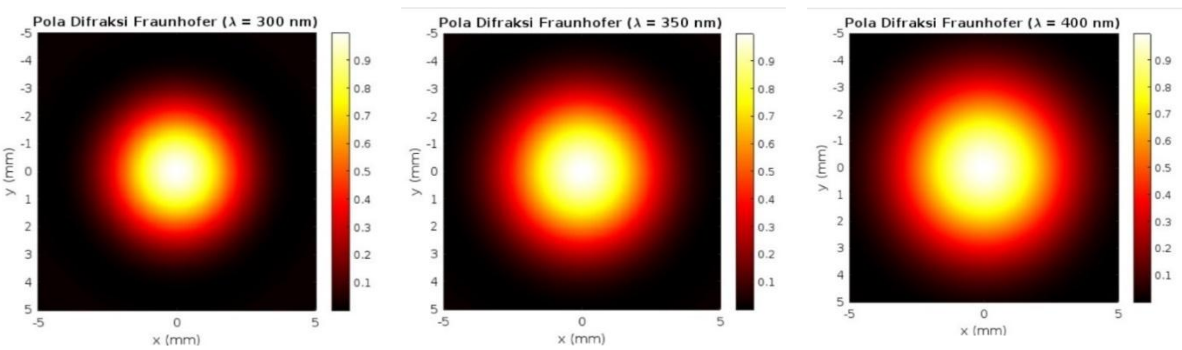
a: radius of circle gap (m)
q: position of the deflected light (m)
R: gap-to-screen distance
I_0: initial light intensity
I: final intensity of the diffraction pattern
J_1: 1st degree Bessel function

The first-order Bessel function can be represented mathematically as:

$$J_1 = \int_0^{\pi} f(\theta)d = \int_0^{\pi} \cos\left(-\frac{kaq}{R}\right) d\theta \quad (4)$$

In the simulation of ring slit frounhofer diffraction using the matlab application, the author compares the effect of wavelength changes on diffraction patterns with the same slit width (radius) but different wavelengths and the effect of radius on diffraction patterns with the same wavelength but different radii. Frounhofer diffraction will occur when the slit light source and the observation layer are far apart (Khachatrian, 2021) (Sun, 2025), Frounhofer diffraction will produce a flat wavefront entering a slit and diffraction pattern so that it becomes a constant shape and intensity (Purnama et al., 2021). Simulation of the application of Frounhofer diffraction can use the Matlab application, because it facilitates a visual understanding of Frounhofer diffraction for circular slits and simplifies the diffraction intensity equation. The use of matlab applications can also make it easier to produce diffraction patterns using the Bessel function (Azlan et al., 2020). Displaying the Fraunhofer diffraction pattern can be done by using a convex lens that is reflected on a screen (Nurdianto et al., 2020). Light passing through an obstacle greatly affects the pattern of bright dark areas produced by Fraunhofer diffraction (Liu et al., 2023). From simulations using the Matlab application, differences in wavelength and radius greatly affect the resulting diffraction pattern (Liu et al., 2022).

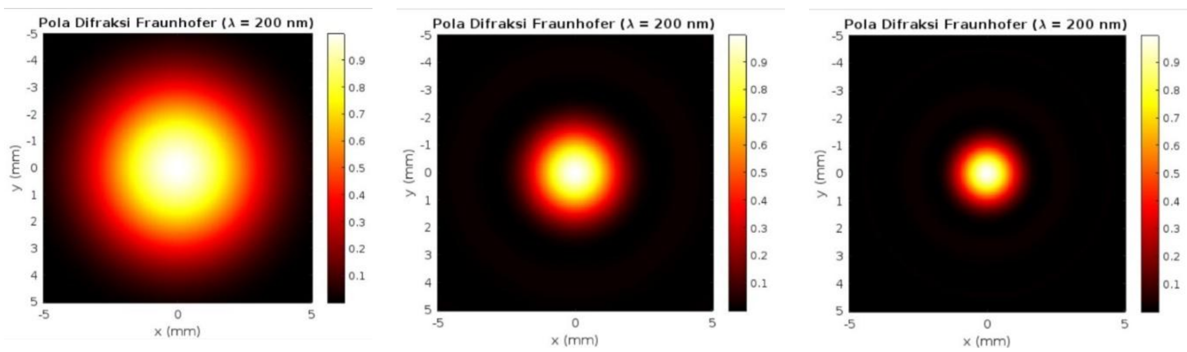
Effect of wavelength change on diffraction pattern with the same slit width (radius) of 0.04 mm:



Wavelength 300 nm | Wavelength 350 nm | Wavelength 400 nm

Figure 3. 0.04 mm radius

Effect of slit width (radius) on diffraction pattern with the same wavelength ($\lambda = 200\ nm$).



Fingers 0,02 | Fingers 0,04 | Fingers 0,06

Figure 4. 200 nm wavelength

In the first simulation, namely analyzing the effect of changes in wavelength on diffraction patterns with a slit width of 0,04 mm and wavelength ($\lambda = 300\ nm$), wavelength $\lambda = 350\ nm$), and wavelength ($\lambda = 400\ nm$) produces a larger circular slit diffraction pattern, or can be said to be directly proportional to the wavelength given, where the greater the wavelength, the larger the circular slit

Frounhofer diffraction pattern, and vice versa, the shorter the wavelength given, the narrower the circular slit Frounhofer diffraction pattern.

This means that wavelength variation affects the diffraction pattern. This phenomenon attests to a property of light waves called diffraction, which occurs when a portion of the wavefront is blocked, either by an obstacle or through one or more slits. As a result, the wavefront undergoes a change in amplitude or phase, which results in distinctive diffraction patterns (Pratidhina et al., 2020). This happens because the ratio of wavelength to slit width increases. As this ratio becomes larger, the intensity distribution of the diffraction pattern also becomes wider. With longer wavelengths, the light waves passing through the slit deflect at larger angles.

In the second simulation, analyzing the effect of the radius influence on the diffraction pattern with a wavelength of ($\lambda = 200\ nm$) and radius 0,02, radius 0,04, and radius 0,06 produces a narrower circular slit diffraction pattern, or can be said to be inversely proportional to the radius given, where the larger the radius given, the narrower the circular slit Frounhofer diffraction pattern will be, and vice versa, the smaller the radius given, the larger the circular slit Frounhofer diffraction pattern will be.

This is because a smaller radius causes the intensity to spread more widely, due to the shorter wavelength, which increases the deflection angle. As the size of the slit decreases the diffraction pattern widens and becomes more pronounced. The far-field diffraction pattern forms a series of concentric rings that vary in shape and size (Solano Navaro et al., 2021). This variation can occur because it is caused by the phenomenon of photoreflection due to the uneven distribution of incident light intensity (Navarro et al., 2021). These findings support previous studies by Solano et al. (2021) that showed concentric ring patterns in far-field diffraction and research by Navarro et al. (2021) who stated that light reflection and uneven distribution of light intensity can cause variations in diffraction patterns.

This research provides an update in the utilization of numerical simulations using Matlab applications in describing the effect of slit size on diffraction patterns with various variations in circular slit size and wavelength. By utilizing the Matlab application, this research successfully developed a method of computing the Bessel function using Matlab with GPU optimization that can improve the efficiency and accuracy of an experiment. However, this research is limited to small to medium values of wavelength and radius and has not tested under extreme or large-scale conditions. Therefore, it is recommended to continue the research by testing this method at larger values and under more complex conditions.

CONCLUSION

This research shows that the Bessel Equation is effective in analyzing Fraunhofer diffraction patterns in circular slits. The use of Matlab with GPU optimization improves the accuracy and efficiency of calculations compared to manual methods. This research also helps understand how the size and shape of the slit affect the diffraction pattern. The findings contribute to the development of optical technology and scientific computing. However, this research is still limited to small to medium values. Further research is recommended to test this method under more complex conditions and applications on a wider scale.

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